Stochastic Processes 2026–27

Problem Set 1

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17 July 2026

Instructions. You may use the axioms (S1–S3, P1–P2), results proved in class, and standard rules of set theory. No other results may be assumed without proof. Venn diagrams are not proofs.

Problem 1. *Let* $(A_n)_{n\geq 1}$ *be a sequence of events in* $\mathcal{F}$*. For each* $n \geq 1$ *define the* tail union

$$C_n := \bigcup_{k=n}^{\infty} A_k.$$

Show that $C_{n+1} \subseteq C_n$ *for every* $n \geq 1$*.*

Problem 2. *Let* $A, B \in \mathcal{F}$ *with* $A \cap B = \emptyset$*. Prove that* $\Pr(A \cup B) = \Pr(A) + \Pr(B)$*.*

Problem 3. *Let* $A_1, A_2, \ldots \in \mathcal{F}$*.*

(a) Prove that for any $A, B \in \mathcal{F}$*,*

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

(b) Using (a), prove by induction on n *that for any* $A_1, \ldots, A_n \in \mathcal{F}$*,*

$$\Pr\left(\bigcup_{k=1}^{n} A_k\right) \leq \sum_{k=1}^{n} \Pr(A_k).$$

(c) Prove that

$$\Pr\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} \Pr(A_k).$$