Test 1 — Stochastic Processes

Maximum Marks: 6 Duration: 30 minutes

Every step must be justified from the axioms of a σ-algebra, the axioms of probability, the defining properties of conditional expectation, a theorem proved in class, or an explicit logical argument written on the side. Steps carry marks, not the final answer.

1. **(3 marks)** Prove that $P(\emptyset) = 0$ from the axioms of probability. Justify every step.

2. **(4 marks)** Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $\mathcal{G} \subseteq \mathcal{F}$ be a sub-σ-algebra. Suppose an integrable, $\mathcal{F}$-measurable random variable X is independent of every $\mathcal{G}$-measurable random variable. Using the definition of conditional expectation, show that

$$E[X \mid \mathcal{G}] = E[X].$$

Justify every step.