DE12: Stochastic Process Internals 2

Madras School of Economics, 10 Marks, 1 hour

Problems

1. [3 marks] For all real x, let

$$F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x).$$

   Prove or disprove: F satisfies the three defining properties of the c.d.f.

2. [2 marks] Consider a continuous Markov chain with two states $S = \{0, 1\}$ with transition probability matrix for any $t \geq 0$ is given by

$$P(t) = \begin{bmatrix} \frac{1}{2} + \frac{1}{2}e^{-2\lambda t} & \frac{1}{2} - \frac{1}{2}e^{-2\lambda t} \\ \frac{1}{2} - \frac{1}{2}e^{-2\lambda t} & \frac{1}{2} + \frac{1}{2}e^{-2\lambda t} \end{bmatrix}.$$

   (a) Find the rate of transition matrix R.

   (b) Write down e^{Rt}.(You don't have to show computations.)

3. Let $N(t)$ be a Poisson process with rate λ. Answer the following:

   (a) [3 marks] Let T_1, T_2 be the first and the second arrival times for the Poisson process $N(t)$ and W_1, W_2 be the first and the second waiting times. For a natural number $n \geq 2$, compute the joint density

$$f_{W_1,W_2|N(t)}(x, y|N(t) = 5).$$

   (b) [2 marks] Find $Cov(N(t_1), N(t_2))$. [Recall $Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$. Also that $Cov(X + X', Y) = Cov(X, Y) + Cov(X', Y)$.]