

DE13: Week 10

1. Let T_1, T_2 be the first and the second arrival times and W_1, W_2 be the first and the second waiting times.

- (a) For a natural number $n \geq 2$, compute the joint density

$$f_{T_1, T_2 | N(t)}(x, y | N(t) = n).$$

- (b) For a natural number $n \geq 2$, compute the joint density

$$f_{W_1, W_2 | N(t)}(x, y | N(t) = n).$$

[Hint: Use the Jacobian method.]

- (c) Compute the marginal densities using the joint densities in part (a).
In other words, compute

$$f_{T_1 | N(t)}(x | N(t) = n), f_{T_2 | N(t)}(y | N(t) = n).$$

- (d) For a natural number $n \geq 2$, compute the conditional density

$$f_{T_1 | T_2, N(t)}(x | T_2 = y, N(t) = n)$$

Check that the density is a pdf.

- (e) Now compute the expectation

$$\mathbb{E}(T_1 | N(t) = n).$$

2. Problems 36, 59, 88 from Chapter 5 of Sheldon Ross's book shared on Google Classroom.