DE13: Week 9

1. Let N be a positive integer valued random variable with finite mean. Suppose $\{X_i \mid i \in \mathbb{Z}_{\geq 0}\}$ is a collection of independent random variables with finite mean that is independent of N. Prove that

$$\mathbb{E}(X_1 + X_2 + \ldots + X_N) = \mathbb{E}(N)\mathbb{E}(X_1).$$

2. Let T be an exponential(λ) random variable. Show that the expectation

$$\mathbb{E}(e^{sT}) = \frac{\lambda}{\lambda - s}$$

for $s \in [0, \lambda)$.

3. Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day.
   (a) What is the expected time until the tenth immigrant arrives?
   (b) What is the probability that the elapsed time between the tenth and the eleventh arrival exceeds two days?

4. Let $N(t)$ be a Poisson process with rate λ. Assume the notations discussed in class. For all real numbers s, t ,and all natural numbers n, calculate the probability

$$\Pr\{T_1 < s \mid N(t) = n\}.$$

This is a conditional cdf. The distribution is a well known one. Interpret the meaning of the answer.

5. Problems 37,44,50 from Chapter 5 of Sheldon Ross's book shared in GCR. (You need problem 1 for some parts)