DE13: Week 7

1. **Experiment**: Suppose a video game store sells game consoles and uses the following policy to restock consoles: If there is 1 or no consoles at the end of the day, the store orders enough to have 3 consoles ready the next day morning. If there is more than one console at the end of the day, the store does not order new consoles. Suppose the game store makes a profit of $Rs.6000$ per console sold but it costs $Rs.1000$ per day to run the store.

   (a) Write a set that serves as a sample space. Remember that a good sample space allows you to answer any question about the problem. It should capture all the information in the description of the experiment.

   (b) Let E be the event that he does not sell any consoles after a week. Write down an element not in E. What is the number of elements in E?

   (c) Construct a sequence of functions on your sample space that serve as the random variables in each of the following case:

      i. N_n is the number of units sold on the nth day.

      ii. I_n is the number of items in the inventory at the end of day n.

      iii. R_n is the number of units restocked at the start of the day n.

   Recall that random variables are functions out of a sample space, i.e. $X : \Omega \to \mathbb{R}$ .So each item in the above list is a function $:\Omega \to \mathbb{R}$. You have to write a formula for each function.

   (d) As a function of N_n, I_n and/or R_n, write a formula for the profit on the nth day.

   (e) Suppose the distribution of demand $\{D_n\}$ for the number of consoles each day is known to be i.i.d process, where each random variable D_n has the p.m.f

   $$f(0) = 0.4, f(1) = 0.3, f(2) = 0.2, f(3) = 0.1.$$

   We have been told that the game store opens with 3 consoles. Among the three stochastic processes $\{R_n, I_n, N_n\}$, how many of them of them have

      i. independent increment property?

      ii. stationary increment proprty?

      iii. Markov property?

2. **Experiment**: Consider a box with 1 red chip and 1 white chip. For every time n, a chip is taken randomly, its color noted, and both this chip and another chip of the same color are put back into the box.

   (a) Write a set Ω that serves as a sample space. Remember that a good sample space allows you to answer any question about the problem. It should capture all the information in the description of the experiment.

   (b) Let W_n denote the number of white chips in the box at the start of time n(before the draw). Write a formula for the function $W_n : \Omega \to \mathbb{R}$.

   (c) Is Markov property satisfied?

   (d) Compute the probability:

   $$\Pr\{W_n = w_n, W_{n-1} = w_{n-1}, W_{n-2} = w_{n-2}, \cdots, W_1 = w_1, W_0 = 1\}$$

   If it looks very abstract, calculate $\Pr\{W_3 = 2, W_2 = 2, W_1 = 1, W_0 = 1\}$ and then think carefully to generalise.

   (e) Is $\{W_n\}$ a time homogenous DTMC?