DE13: Week 5

BA2022, Stochastic Process, MSE

1. In class we showed that $N = (I - Q)^{-1}$ for a markov chain with absorbing states where Q is the block of the t.p.m containing only transient states and N is the the matrix of expected number of visits. By rearranging the equation (and using definition of inverse), we will get

$$N = I + NQ.$$

Prove this identity directly using first step analysis.

2. The transition probability matrix for a 5 state markov chain with row, column index set $0, 1, 2, 3, 4$ (in that order) is

$$\tilde{P} = \begin{bmatrix} 1 & 0 & 0 & 0.1 & 0.3 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.2 & 1 & 0.8 & 0.7 \\ 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

(a) Draw the graph and list the absorbing and transient states. Reorder the states to bring it to the form:

$$P = \begin{bmatrix} Q & 0 \\ R & I \end{bmatrix}$$

(b) Given that the process starts in state 1, calculate the expected number of visits to state 4 using the matrix method and also using first step analysis.Quote the theorem correctly!

(c) If the process hits one of the absorbing states at time n, we will say that the **age of the process** is n.What is the expected age of the process if the process starts at state 4? [Hint: Either do first step analysis or use N matrix.]

(d) Given that the process starts in state 1, what is the probability that the process hits state 0?

3. A motor insurance company puts policy holders into three categories:

(a) No discount on premiums (state 1)

(b) 20% discount on premiums (state 2)

(c) 30% discount on premiums (state 3)

(d) 40% discount on premiums (state 4)

New policy holders start with no discount (state 1). Following a year with no insurance claims, policy holders move up one level of discount. If they start the year in state 4 and make no claim, they remain in state 4. Following a year with at least one claim, they move down one level of discount. If they start the year in state 1 and make at least one claim, they remain in state 1. The insurance company believes that probability that a motorist has a claim free year is 80% .

(a) Model the states of a random policy holder as a Markov chain. Write the t.p.m and the state graph.

(b) Compute f_{44} . Let T denote the time taken for the process to return to 4 for the first time. If the process starts from 4, what is the average value of T?

(c) In the long run, can you guess the fraction of time that a policy holder enjoys 40% discount? [Hint: Part b is telling you the answer.]