# DE13: Week 4.

**BA2022, Stochastic Process, MSE**

1. If $\{X_n\}$ is a time homogenous DTMC then show that $\Pr\{X_2 \neq 1, X_3 \neq 1, \ldots, X_{n-1} \neq 1, X_n = 1 \mid X_1 = 0\} = f_{10}^{(n-1)}$.

2. Manish and Medha bet 1 hundred rupees in each round. In other words, Medha wins 1 hundred rupees each round with probability $p$ and loses 1 hundred rupees with probability $1-p$. Assume that Manish starts the game with 1 hundred rupees and Medha starts with 2 hundred rupees. The game ends when one of them goes bankrupt. Let $M_n$ denote Medha's fortune (in units of hundred rupees) at the end of round $n$ and let $M_0$ denote Medha's initial fortune.

   (a) What is the range of the variable $M_n$ for any $n$?

   (b) Show that $M_n$ is a DTMC. Draw the state diagram of the Markov chain.

   (c) In terms of the notations in the class, what is $f_{33}$?

   (d) Calculate $f_{23}$ and $f_{31}$, use the following technique: For now ignore that $M_0 = 200$. Let $\mathcal{A}$ denote the event "Medha will have 3 hundred rupees at some time". Show that

   $$\Pr(\mathcal{A} \mid M_0 = 200) = \Pr(\mathcal{A} \mid M_0 = 100)(1-p) + p.$$

   $$\Pr(\mathcal{A} \mid M_0 = 100) = \Pr(\mathcal{A} \mid M_0 = 200)p.$$

   [Hint: Marginalize over $M_1$, use Markov property and then use the idea in Problem 1.]

   (e) Plug in

   $$f_{32} = \Pr(\mathcal{A} \mid X_0 = 200),\ f_{31} = \Pr(\mathcal{A} \mid X_0 = 100)$$

   in part (c) and solve the system of equations. Compute $f_{32}, f_{31}$.

   (f) What is the probability that Medha bankrupts Manish if she starts with 2 hundred rupees and Manish starts with 1 hundred rupees?

   (g) Let $T$ be the number of rounds played. Let $u_1 = \mathbb{E}(T \mid X_0 = 1)$ and $u_2 = \mathbb{E}(T \mid X_0 = 2)$. Using the technique employed in part (d), show that

   $$u_1 = pu_2 + 1,\ u_2 = (1-p)u_1 + 1.$$

   Now compute $\mathbb{E}(T)$ using $u_1$ and $u_2$.As an alternative, directly compute $\mathbb{E}(T)$ by analyzing the chain as well.

3. Using the technique in parts (d) and (e) above, compute the probability that you will win 5000 rupees (starting from 2000 rupees) in the problem discussed at the start of the chapter.