DE 13: Week 3

1. Suppose a video game store sells game consoles and uses the following policy to restock consoles: If there is 1 or no consoles at the end of the day, the store orders enough to have 3 consoles ready the next day morning. If there is more than one console at the end of the day, the store does not order new consoles. Suppose the distribution of demand $\{D_n\}$ for the number of consoles each day is known to be i.i.d process, where each random variable D_n has the p.m.f

$$f(0) = 0.4, f(1) = 0.3, f(2) = 0.2, f(3) = 0.1.$$

Let I_n denote the number of consoles in the inventory at the end of day n. Clearly $I = \{I_n\}$ is a discrete time process. Further suppose the game store makes a profit of $Rs.6000$ per console sold but it costs $Rs.1000$ per day to run the store. We have been told that the game store opens with 3 consoles.

   (a) Prove that I is a DTMC. [Hint: Write I_{n+1} in terms of I_n and D_n.]

   (b) Write down the one-step t.p.m and state transition graph for this DTMC. Is the DTMC irreducible?

   (c) What is the expected profits for the first two days?

   (d) Compute the stationary distribution of the process.Estimate the stabilized average profit per day.

   (e) [Computer calculation, optional] Compute P^{10000} . Using the given initial distribution and P^{10000}, check if the distribution I_{10000} is close to the stationary distribution.

2. Let X denote a DTMC with set of states S. For a state $i \in S$ and a natural number n, consider the expression

$$r_i(n) = \Pr\{X_1 \neq i, X_2 \neq i, \cdots X_{n-1} \neq i, X_n = i \mid X_0 = i\}.$$

   (a) Describe $r_i(n)$ in words.

   (b) Let $\{X_n\}$ be a collection of iid Bernoulli(p) random variables. As we proved in class, X is a DTMC. Compute $r_i(n)$ for $i = 0, 1$ for this process.What is the value of $\sum_{n=0}^{\infty} r_i(n)$?

3. Let $\{X_n\}$ and $\{Y_n\}$ be two independent DTMCs, with only two states, that share the same t.p.m

$$\begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}.$$

Consider a new stochastic process Z created using a fair coin as follows: At every instant of time, you toss the coin, if the coin shows a head, then set $Z_n = X_n$, otherwise set $Z_n = Y_n$.

   (a) Show that Z is a DTMC.

   (b) Calculate the one-step t.p.m of Z.

   (c) What is the stationary distribution of Z?

   (d) Compute $r_0(n)$,$r_1(n)$ (defined in problem 2)for Z.What is the value of $\sum_{n=0}^{\infty} r_i(n)$?