DE 13: Assignment 2

1. Let $\{X_n\}$ is a DTMC with values in $\{1, 2, 3, ...\}$. If P is the t.p.m of the DTMC and f_{X_n} is the p.m.f of the random variable X_n for all naturals n, then show that

$$Pf_n = f_{n+1}.$$

where the ith coordinate of vector f_k is the value of the p.m.f $f_{X_k}(i)$. [Hint: Total probability theorem.]

2. Let the state space of a time homogenous **DTMC** $\{X_n\}$ be $\{0, 100, 200\}$. The one-step t.p.m is given by

$$P = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.33 & 0 & 0.67 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$$

   (a) Draw the graph that represents these transitions.

   (b) Compute the two step transition probability.

   (c) If you started from state 0 at time 0. Compute $\mathbb{E}(X_2)$.

   (d) Find a p.m.f f_{X_0} for the X_0 with the property

$$f_{X_1} = f_{X_0}.$$

   For this value of f_{X_0}, what is the value of $f_{X_{2024}}$?

   (e) Compute the joint p.m.f of X_1, X_2 assuming $X_0 = 0$.

3. Suppose there are three types of laundry detergent, 1, 2, and 3, and let X_n be the brand chosen on the nth purchase. Customers who try these brands are satisfied and choose the same thing again with probabilities $0.8, 0.6$, and 0.4 respectively. When they change they pick one of the other two brands at random.

   (a) Write the t.p.m and the state transition graph.

   (b) If the market shares of these brands stabilize over time, what is the eventual market share of these brands?