DE13: Week 12

1. Let $T \sim$ Exponential(λ), then prove

$$\Pr\{T > t + s \mid T > t\} = \Pr\{T > s\}.$$

   This property is called the memoryless property of an exponential random variables. Interpret the formula in terms of waiting time of an auto.

2. *Probability/Calculus practice*: Let T, S be independent exponential random variables with parameters $\lambda = 5$ and $\mu = 10$ respectively. Compute the following probabilities using integration over the joint pdf:

   (a) $\Pr\{2T > S\}$

   (b) $\Pr\{\min\{T, S\} < 5\}$

   (c) $\Pr\{4T + 3S < 10\}$

3. Consider a machine that works for an exponential distributed amount of time having mean $1/5$ before breaking down; and suppose that it takes an exponential amount of time having mean $1/10$ to repair the machine. Modelling the working state of the machine as a two state pure jump CTMC $X(t)$, and if the machine is in working condition at time $t = 0$, then what is the probability that 7 jumps have occurred before $t = 10$?