DE 13: Assignment 1

Definition. A stochastic process $\{X_t\}_{\{t\in T\}}$ where $T = \mathbb{R}$ or $\mathbb{Z}$ is said to have stationary increment property if for any numbers t, h in T, the distribution of $X_{t+h} - X_t$ is independent of t and only depends on h.

This problem set will produce examples and non-examples of processes with stationary increment property and independent increment property.

Homework Problems

1. A biased coin which lands heads with probability p is tossed in succession forever. Let N_k denote the number of tails in the kth toss for every natural number k.

   (a) [5 marks] Let k be a natural number. Identify the distribution of N_k.

   (b) [10 marks] Compute the distribution of $N_{15} - N_{10}$. Let k, h be natural numbers. Does the distribution of $N_{k+h} - N_k$ depend on k? Explain. Does the process N have stationary increment property?

   (c) [10 marks] Compute the probabilities

   $$\Pr\{N_3 - N_2 = 1 \mid N_2 - N_1 = 1\} \text{ and } \Pr\{N_3 - N_2 = 1\}.$$

   Does that the stochastic process N have independent increment property?

2. A coin is tossed in succession forever. Let N_k denote the number of tails in the kth toss. Define

   $$S_k = N_1 + N_2 + \cdots + N_k.$$

   (a) [10 marks] What is the distribution of $N_1 + N_7 + N_{19} + N_{323}$? In general, let $k_1, k_2, \cdots, k_n$ denote any collection of n natural numbers, then what is the distribution of $N_{k_1} + N_{k_2} + \ldots + N_{k_n}$? [Hint: It's a well known distribution.]

   (b) [10 marks] Does the distribution of $S_{k+h} - S_k$ depend on k? In other words, does the process S have stationary increment property?

   (c) [5 marks] In class, we discussed that the process S has independent increment property. Write down the proof discussed in class.