Time Series Analysis - Homework 3

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1. Differentiate the infinite geometric series formulae twice to obtain the following formulae: For $|x| < 1$
$$1 + \binom{3}{2}x + \binom{4}{2}x^2 + \binom{5}{2}x^3 + \cdots = \frac{2}{(1-x)^3}.$$

2. Let L denote the lag operator, $Lx_t = x_{t-1}$.

   (a) $x_t - 0.6x_{t-1} = u_t$,

   (b) $x_t - 1.2x_{t-1} + 0.32x_{t-2} = u_t$,

   (c) $8x_t - 12x_{t-1} + 6x_{t-2} - x_{t-3} = 8u_t$.

   (a) Rewrite each difference equation using lag–operator notation.

   (b) For each equation above, write the associated lag polynomial. Factor the lag polynomial whenever possible.

   (c) Solve for x_t in terms of current and past values of $\{u_t\}$ whenever the solution exists.

   (d) Plot the impulse response.

3. Consider the commodity market model studied by Muth (1961). In this model, firms hold inventories based on expected future prices, so current decisions may depend on anticipated prices in the next period. Time is indexed by integers, $t \in \mathbb{Z}$.

   Let p_t denote the price at time t, C_t consumption demand, Y_t output, I_t inventories, and p_t^e the expected price. Let $\{X_t\}$ be a bounded exogenous sequence.

   The economy is described by:
$$\begin{aligned} C_t &= -\beta p_t, \qquad \beta > 0, \\ Y_t &= \gamma p_t^e + X_t, \qquad \gamma > 0, \\ I_t &= \alpha(p_{t+1}^e - p_t), \qquad \alpha > 0, \\ Y_t &= C_t + (I_t - I_{t-1}). \end{aligned}$$

   Assume perfect foresight:
$$p_t^e = p_t \quad \text{for all } t.$$

   Note that the model involves both lagged and forward values of price. (You may define the forward operator by $L^{-1}p_t = p_{t+1}$.)

(a) Derive the difference equation governing the price sequence $\{p_t\}$.

(b) Write the resulting equation using lag–operator notation involving L and L^{-1}.

(c) Solve the difference equation and express p_t in terms of $\{X_t\}$ and arbitrary constants.

(d) State conditions under which $\{p_t\}$ remains bounded for all bounded sequences $\{X_t\}$. Use these conditions to determine the constants.