Time Series Analysis - Homework 1

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1. Consider the difference equation

$$m_t = 0.72\, m_{t-1} + w_t, \qquad m_{-1} = 0.$$

```python
import numpy as np
import matplotlib.pyplot as plt

T = 60
phi = 0.72

w = np.zeros(T)
w[0] = 0.01

# w = np.zeros(T)
# w[:] = 0.01

# w = np.zeros(T)
# w[0:3] = 0.01

m = np.zeros(T)
for i in range(T):
    if i == 0:
        m[i] = w[i]
    else:
        m[i] = phi * m[i-1] + w[i]

plt.plot(m)
plt.plot(w, linestyle="--")
plt.xlabel("t")
plt.show()
```

   (a) Run the code and describe the sequence $\{w_t\}$.

   (b) Explain how the sequence $\{m_t\}$ is generated from $\{w_t\}$.

   (c) Uncomment each alternative definition of w (one at a time), rerun the code, and explain how the output changes.

2. * Goldfeld's estimated money–demand equation is

$$m_t = 0.27 + 0.72\, m_{t-1} + 0.19\, I_t - 0.045\, r_{bt} - 0.019\, r_{ct}.$$

Assume interest rates are constant and $m_{-1} = 0$. Suppose log income increases permanently by 1% at time $t = 0$, i.e.

$$\Delta I_t = 0.01 \quad \text{for all } t \geq 0.$$

   (a) Write down the difference equation satisfied by Δm_t.

   (b) Solve this difference equation and compute

$$\lim_{t \to \infty} \Delta m_t.$$

   (c) The *long-run income elasticity of money demand* is defined as the limiting percentage change in money holdings resulting from a permanent 1% increase in income. Compute this elasticity using your answer above.