DE09:Time series analysis: Internals-2 (Retake)

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Rules of the exam

1. 90 minutes, 22 marks paper. Maximum score is 20.
2. Anyone in posession of mobile/smartwatch or any electronic device capable of connecting to a network will be given a zero.
3. You are allowed to use definitions, theorems and formulae proved in class. Any other methods will not score marks allowed unless you prove all the relevant claims implicit in your method.
4. All the questions follow notations used in class. You may use a calculator.

Problems

1. [4 marks] Derive the infinite sample forecast for an AR(1) process using the Wiener-Kolmogorov representation.
2. [3 marks] Given that the time series $y = [1, 1, 2, 1, 1, 3]$ is drawn from an AR process $y_t = \phi y_{t-1} + \epsilon_t$ with $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$
   (a) Compute the MMSE estimate of ϕ.
   (b) Compute the MMSE estimate of σ^2.
3. [3 marks] Using the lagged Y representation of Weiner Kolmogorov representation, write the optimal forecast $\hat{Y}_{t+1}$ if the time series follows a process
$$Y_t = 0.5Y_{t-1} + \epsilon_t + 2\epsilon_{t-1}$$
   where $X_t = \{Y_\tau : \tau \leq t\}$ for all t.