DE09:Time series analysis: Internals-1

Srikanth Pai, Madras School of Economics.

Rules of the exam

1. 90 minutes, 22 marks paper. Maximum marks you can score is 20.
2. Anyone in posession of mobile/smartwatch or any electronic device capable of connecting to a network will be given a zero.
3. You are allowed to use definitions, theorems and formulae proved in class. Any other methods will not score marks allowed unless you prove all the relevant claims implicit in your method.
4. All the questions follow notations used in class. You may use a calculator.

Problems

1. [4 marks] Write a note on MA(∞) process and clearly outline the assumptions that go into proving AR(1) is MA(∞).
2. [4 marks] If $\hat{Y}_{t+1}$ is the minimum mean squared error forecast given $\mathbf{X}_t$ then prove that $\hat{Y}_{t+1} = \mathbb{E}(Y_{t+1}|\mathbf{X}_t)$.
3. [4 marks] The basic form of the Samuelson model is (in units of billions of dollars and time unit is quarters):
$$Y_t = C_t + I_t + G_t$$
where: Y_t is the national income at time t, C_t is consumption, I_t is investment, G_t is government spending (assumed constant for simplicity). Consumption depends on past income:
$$C_t = cY_{t-1}$$
where c is the marginal propensity to consume. Investment depends on changes in income:
$$I_t = I_0 + b(Y_{t-1} - Y_{t-2})$$
where: I_0 represents autonomous investment, b is the accelerator coefficient. Assume $c = 0.6, I_0 = 20, b = 0.5, G_t = 50$.
   (a) Derive a second order difference equation for national income. Write it in lag operator format.
   (b) Comment on stability and cyclicity of national income. Compute the period of the cycle if the system is cyclic.
4. [4 marks] Consider a $MA(1)$ process
$$Y_t = 1 + \epsilon_t + 0.5\epsilon_{t-1}$$
where $\{\epsilon_t\}$ is white noise with unit variance. Work out the optimal linear forecast based on two latest samples. The answer should be in the form $aY_t + bY_{t-1} + c$ where a, b, c are real numbers that you will have to find.

5. [2 marks] The second order difference equation $y_t = y_{t-1} + y_{t-2} + w_t$ can be written as

$$\underline{\xi_t} = \mathbf{F}\underline{\xi_{t-1}} + \underline{v_t}$$

in matrix format as done in class.Compute $\mathbf{F}$ and calculate the dynamic multiplier $\dfrac{\partial y_{t+2}}{\partial w_t}$.

6. [2 marks] If $X_t = W_t - tW_{t-2}$, where $\{W_t\}$ are independent random variables and they are distributed according to the uniform distribution in the interval $[0, 1]$, is the process X_t covariance stationary? Justify using calculations.

7. [2 marks] Goldfeld's (1973) estimated money demand function for the United States. Goldfeld's model related the log of the real money holdings of the public (m_t) to the log of aggregate real income (I_t), the log of the interest rate on bank accounts (r_b), and the log of the interest rate on commercial paper (r_c):

$$m_t = 0.27 + 0.72m_{t-1} + 0.19I_t - 0.045r_{bt} - 0.019r_{ct}.$$

Suppose there is a permanent increase of 5% in aggregate real income, and assume that there is a constant interest rate of 2%. What is the long run effect on the present value of money demand?