MADRAS SCHOOL OF ECONOMICS

UNDERGRADUATE PROGRAMME IN ECONOMICS (HONOURS) [2022-25]

SEMESTER 6 [JANUARY – MAY, 2025]

REGULAR EXAMINATION, MAY 2025

Course Name: Introduction to Time Series Analysis **Course Code:** DE09

Duration: 2 Hours **Maximum Marks: 60**

Instructions: *For part A write short answers (most preferably a couple of sentences). For part B, writing relevant formulae and quoting correct definitions helps me give you part marks. For questions with qualitative answers, creative writing is prohibhited.*

Part A: Answer all questions (1 mark x 10 questions = 10 marks)

1. What is a covariance stationary process?
2. If
$$y_t = 0.3y_{t-1} + 0.2y_{t-2} + \epsilon_t$$
where ϵ_t is white noise, what is the value of
$$\mathbb{E}\left(y_{t+1} \mid y_t = 1, y_{t-1} = 2, y_{t-2} = 3, \cdots\right)?$$
3. If the eigenvalues of a second order recurrence are $1+i, 1-i$, what is the period of the time series?
4. What is a unit root ARMA process?
5. Give an example of a time series for which the ACF and PACF vanishes within finite number of lags. No proof needed.
6. Suppose the time series $\{y_t\}$ satisfies the equation
$$y_t = \epsilon_t + 0.2\epsilon_{t-1}$$
where $\{\epsilon_t\}$ is Gaussian white noise. Compute the covariance $\text{Cov}(y_{2025}, y_{2026})$.
7. What is the use of recursive substitution in linear difference equations?
8. Find the optimal linear forecast $\hat{Y}_{t+1}$ if the present and past innovations $\{..., \epsilon_{t-1}, \epsilon_t\}$ are observed and the time series is an MA(1) process given by
$$Y_t = \epsilon_t + 0.3\epsilon_{t-1}.$$
9. Is the process
$$Y_t = \epsilon_t + 3\epsilon_{t-1}$$
invertible? Explain
10. If the p-value computed for a Dickey-Fuller test is 0.00001, what will you conclude about the time series? Explain.

Part B: Answer any five. (10 mark x 5 questions = 50 marks)

11. Explain all the steps of the Box Jenkins methodology systematically motivated by all the theorems taught in this course. Write down rigorous descriptions of every step and describe all possible equations used at each step. [Note that this involves *stating* almost all the concepts taught in the course, but in the sequence of data analysis.]

12. Suppose that you have estimated the first five correlation coefficients using a series of length 100 observations and found them to be

Lag	1	2	3	4	5
AR coefficient	0.207	-0.013	0.086	0.005	-0.022

   (a) [4 marks] Test each of the individual coefficients for significance.

   (b) [6 marks] Write down the Box-Pierce and Ljung-Box test statistics to test all five coefficients jointly.

13. Answer the following questions:

   (a) [3 marks] Construct an example of a stationary but non-ergodic (in the mean) process. Prove your claims.

   (b) [4 marks] State the Wiener Kolmogorov representation for observed data (lagged Ys) and apply it an AR(1) process to get an infinite sample two step ahead optimal forecast.

   (c) [3 marks] Let $y_t = \rho y_{t-1} + \epsilon_t$. Suppose $\rho = 1$, we have observed T samples and $\hat{\rho}_T$ is the OLS estimate of ρ.Compute the probability

$$\Pr\left\{\lim_{T\to\infty} T(\hat{\rho}_T - 1) < 0\right\}$$

   and conclude that the distribution of the statistic is skewed. Justify your answer carefully. The limiting random variable $\lim_{T\to\infty} T(\hat{\rho}_T - 1)$ need not be derived but you should clearly explain all the variables.

14. Answer the following question.

   (a) [2 marks] State the Yule-Walker equations satisfied by the autocorrelation functions. of an $AR(p)$ process.

   (b) [4 marks] If $y_t = (0.5)^t$ for $t = 1, 2, 3, 4, 5,$ is *believed* to be drawn from a zero mean $AR(2)$ process, estimate the coefficients of $AR(2)$ process using Yule Walker equations.

   (c) [4 marks] Suppose a time series for y_t satisfies the following recurrence

$$y_t = 3y_{t-1} + 7y_{t-2} + 0.1\epsilon_t + 0.8\epsilon_{t-1}$$

   is written in $MA(\infty)$ form:

$$y_t = \sum_{i=0}^{\infty} \psi_j \epsilon_{t-j}$$

   What is the value of ψ_3 ? What is the long run multiplier?

15. Answer the following question.

   (a) [3 marks] Prove: Let $\{Z_t\}$ satisfy the equation:

$$(1 - \phi L^4)Y_t = \epsilon_t$$

   where $|\phi|$ is less than 1 and $\{\epsilon_t\}$ is iid $\mathcal{N}(0, 1)$. Then the ACF for $\{Y_t\}$ is equal to zero if lags are not in multiples of four.

   (b) [5 marks] (Essay) Write a note on AIC critieria and explain its motivation that arises from information theory.

   (c) [2 marks] Consider the stationary ARMA(1, 1) process given by

$$Y_t = 0.5Y_{t-1} + \epsilon_t - 0.3\epsilon_{t-1}$$

   Argue that it has a $MA(\infty)$ representation.

16. Answer the following questions:

   (a) [2 marks] Draw the rough sketch of a random walk and a covariance stationary zero mean process.

   (b) [4 marks] A time series $\{Y_t\}$ is given by

   $$Y_t = P\cos(t) + Q\sin(t)$$

   for integers t, where P and Q are iid $N(0, \sigma^2)$. Calculate

   $$Cov(Y_t, Y_{t+h})$$

   for $h = 0, \pm 1, \pm 2, \cdots$. Is $\{Y_t\}$ a covariance stationary process? [Hint: Remember $\cos(A - B) = \cos A \cos B + \sin A \sin B$.]

   (c) [4 marks] Let $y_t = y_{t-1} + \epsilon_t$ where ϵ_t is (standard) normally distributed white noise. Show that the distribution of

   $$\lim_{T\to\infty} \frac{1}{T} \sum_{t=2}^{T} \epsilon_t y_{t-1}$$

   is $\frac{1}{2}(W(1)^2 - 1)$ where $W(t)$ is a Wiener process. You may assume the functional central limit theorem.

17. Let y_t satisfy p-th order homogenous linear recurrence, then

   (a) [3 marks] Show that the equations can be written in a matrix form

   $$\mathbf{y}_t = F\mathbf{y}_{t-1} + \mathbf{v}_t$$

   where $\mathbf{y}_t$ is a vector of the latest p values of the time series y.

   (b) [3 marks] Assume the time series is stationary if eigenvalues of F are strictly inside the unit circle. Show that $I - F$ is invertible if the time series is stationary.

   (c) [4 marks] Show that the long run multiplier is the $(1,1)$ entry of the matrix $(I - F)^{-1}$ for a stationary time series.