

Assignment 4: Estimation of parameters

DE09 Time series analysis, Srikanth Pai, MSE

1. Consider the AR(1) model:

$$y_t = \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

for $t = 1, \dots, T$.

- (a) Write down the likelihood function for the sample $\{y_t\}_{t=1}^T$.
- (b) Derive the log-likelihood function.
- (c) Compute the first-order conditions for MLE estimation of ϕ and σ^2 .
- (d) Show that the MLE estimator for ϕ is:

$$\hat{\phi} = \frac{\sum_{t=2}^T y_t y_{t-1}}{\sum_{t=2}^T y_{t-1}^2}$$

- (e) If $|\phi| < 1$, is the distribution of $\hat{\phi}$ as $T \rightarrow \infty$?
- (f) If $\phi = 1$, what happens to distribution of $(\hat{\phi} - 1)$ as $T \rightarrow \infty$?

2. Consider the MA(1) process:

$$y_t = \varepsilon_t + \theta \varepsilon_{t-1}, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

- (a) Write down the likelihood function assuming y_1, \dots, y_T are observed.
- (b) Derive the log-likelihood function.
- (c) Discuss why MLE estimation of θ in MA(1) is challenging.

3. ACF and PACF

- (a) Define ACF and show that ACF vanishes within p terms for a MA(p) process.
- (b) Define PACF and show that PACF vanishes within q terms for a AR(q) process.

4. Subjective questions

- (a) Explain why gradient descent may struggle to estimate the parameters of an MA(1) model.
- (b) What is conditional likelihood function? Why do we use it?
- (c) What is the method of moments? Can this be used instead of MLE method?