Time series: Assignment 2

Srikanth Pai, Asst. Prof, MSE

1. Try to write clear proofs for "Show that.." questions. Your proofs must start by stating definitions, followed by a sequence of logical inferences and the conclusion must be at the end.
2. For "Give an example...." questions, state the example clearly at the start and then justify why it works afterwards.
3. I do not condone using AI tools blindly. Use it to learn intermediate steps and you will be prepared for the exam.

Maths practice

1. If $\{W_t\}$ is white noise, compute the following
   (a) $\mathbb{E}(W_t|W_t = 2), \mathbb{E}(W_{t+1}|W_t = 2)$
   (b) $\mathbb{E}(W_t W_{t+1})$
   (c) $Cov(W_t + 5, W_{t+5})$
   (d) $Cov(7W_t + 6W_{t+2}, 5W_t + 3W_{t+1})$
2. If $Z_t = 3W_t + 2W_{t+1} + 4$, then compute
$$\lim_{T\to\infty} \frac{Z_1 + Z_2 + \cdots + Z_T}{T}.$$
   Can you apply law of large numbers to Z_t? Justify your answer.

Stationarity and Ergodicity practice

1. Show that any i.i.d process is stationary.
2. Show that any stationary process is covariance stationary.
3. Give an example of a stationary process that is not ergodic.
4. For each of the following, state if it is a covariance-stationary process. If so, check if it is ergodic (in the mean). Here, $\{W_t\}$ is i.i.d. $N(0, 1)$.
   (a) $X_t = 10000 + W_t - W_{t-3}$

(b) $X_t = W_3$

(c) $X_t = t + W_3$

(d) $X_t = W_t^2$

(e) $X_t = W_t W_{t-2}$

Check if a, b, c, d are stationary as well.

5. If $X_t = W_t - 2W_{t-2}$, where $\{W_t\}$ are independent random variables and they are distributed according to the uniform distribution, is the process X_t covariance stationary?

6. Calculate the mean and autocorrelation functions of $MA(2)$ process and show that the process is covariance stationary and ergodic in the mean. [You cannot use the general $MA(q)$ formula unless you prove the general formula (which is considerably harder to do than $MA(2)$).]

7. Write a note on $MA(\infty)$ process and clearly outline the assumptions that go into proving $AR(1)$ is $MA(\infty)$.

Applications

1. A fair coin is tossed every second. Your score at the 't'th second is the average number of heads in the latest four tosses.

   (a) Model the score as a MA(q) process. What is q? Is it covariance stationary and ergodic?

   (b) Model the score as a AR(p) process.What is p?Is it covariance stationary and ergodic?

2. A researcher estimated the following relationship for the inflation rate (π_t):

$$\pi_t = -0.05 + 0.7\pi_{t-1} + 0.6\pi_{t-2} + \epsilon_t$$

   (a) Suppose that in periods 0 and 1, the inflation rate was 10 and 11%, respectively. Find the general solutions for the inflation rate.

   (b) Discuss the shape of the dynamic multiplier. Given that the United States is not headed for runaway inflation, why do you believe that the researcher's equation is poorly estimated?