Time series: Assignment 1

Srikanth Pai, Asst. Prof, MSE

Maths practice

1. If 1 is a root of the polynomial $x^5 - 3x^4 + 10x^3 - 11x^2 - 2x + k = 0$, what is the value of k?
2. Root-Coefficient relations:
   (a) If the roots of the quadratic $ax^2 + bx + c = 0$ are real, then what is the condition on the roots?
   (b) If the roots of the quadratic $ax^2 + bx + c = 0$ have absolute value less than 1, then what is the condition on the roots?
3. Suppose (b, c) represents a generic point in the coordinate plane. If each point in the plane represents a quadratic $x^2 + bx + c = 0$. Shade the regions where
   (a) quadratic equations have all roots with absolute values less than one and roots are real.
   (b) quadratic equations have all roots with absolute values less than one and roots are complex.
   (c) quadratic equations have at least one root whose absolute value bigger than one.
4. Suppose $|\phi_1| + |\phi_2| + ... + |\phi_n| < 1$ then show that all the roots of the polynomial $x^n - \phi_1 x^{n-1} - \phi_2 x^{n-2} \cdots - \phi_n = 0$ have absolute value less than 1.
5. If $P_t = \phi P_{t-1} + W_t$, (all capital variables are random variables and W_t are all standard normal random variables):
   (a) Compute conditional expectation $\mathbb{E}(P_t | P_{t-1} = p_{t-1})$.
   (b) If $P_{-1} = 1$, compute expectation $p_t = \mathbb{E}(P_t)$.
   (c) Find a difference equation for the means p_t and solve it.

Time series practice

1. If $y_t = 2y_{t-1} + w_t$, $y_{-1} = 0$ and $w_t = 1$ for all t.
   (a) Find the values of y_0, y_1, y_2 by directly plugging the values into the difference equation.
   (b) Use recursive substitution formula to find a formula for y_t.
   (c) Compute the dynamic multiplier.
   (d) Is the time series stable? Is the time series invertible?
2. Suppose the present value of a time series process is given by the usual discounting formula. Compute the dynamic multiplier in this case. Work it out for our proposed model of momentum based investing (we discussed it in class).

3. In practice we are often interested in the "rate of change" of a variable. In time series, we implement it by looking at the difference time series. If $\{y_t\}$ is a time series, the difference series is defined as

$$d_t = y_t - y_{t-1}.$$

   (a) Suppose d satisfies a first order difference equation: $d_t = \phi d_{t-1} + w_t$. Then show that y_t satisfies a second order difference equation.

   (b) Show that if

      i. $|\phi| < 1 \implies y_t$ is stable.

      ii. Under what conditions does y_t show cyclic behaviour?

4. A difference equation whose characteristic equation contains 1 as a root is called a "unit root process".

   (a) If

$$y_t = 3y_{t-1} - 7y_{t-2} + 2y_{t-3} + ky_{t-4} + w_t$$

   is a unit root process, then what is the value of k?

   (b) Is it true that the "difference time series" from the previous problem is always a unit root process?

5. We have a first order difference equation, with our usual notation. If it is not required that the time series be bounded, and if $|\phi| < 1$, then show that there are multiple solutions to difference equation. [Hint: Show that $K\phi^t + w_t + w_{t-1}\phi + w_{t-2}\phi^2 \cdots$ is a solution for any value of K.]

Economics/Finance applications

1. The following model was originally developed to explain the volatility in agricultural prices, let the market for a product—say, wheat—be represented by

$$d_t = a - \gamma p_t, \quad \gamma > 0$$

$$s_t = b + \beta p_t^* + \epsilon_t, \quad \beta > 0$$

$$s_t = d_t$$

   where

   d_t = demand for wheat in period t ,

   s_t = supply of wheat in t ,

   p_t = market price of wheat in t ,

   p_t^* = price that farmers expect to prevail at t,

   ϵ_t = a zero mean stochastic supply shock, and parameters a, b, γ, and β are all positive such that $a > b$.

   Unlike the actual market for wheat, the model ignores the possibility of storage. *The essence of the cobweb model is* that farmers form their expectations in a naive fashion, let farmers use last year's price as the expccted market price.

   (a) If the system is stable and the prices converge to a fixed value in the long run, what would be that value?

   (b) Form a reduced equation for prices and deduce the condition on parameters that will produce stable prices.

(c) Compute the dynamic multiplier of prices w.r.t to supply shock.

2. Revisit Naveen Sir's tutorial 3 and 4 (uploaded in GCR). In each case, work out the reduced equations. Compute the dynamic multiplier. Comment on stability, cyclicity and compute the period of the cycle in case of cyclicity.You should be able to answer how the time series will change if an autonomous variable is changed in a quarter or permanently changed.