Lecture 03: Counting Methods and Random Variables

BA II Year, Madras School of Economics
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Due date: TBA

Textbook: Larsen & Marx, *An Introduction to Mathematical Statistics and Its Applications*, Fifth Edition.

Topics: Multiplication rule, permutations, combinations, combinatorial probability, introduction to random variables (Sections 2.6–2.7, 3.1).

1. Suppose that the format for license plates in a certain state is two letters followed by four numbers.
   (a) How many different plates can be made?
   (b) How many different plates are there if the letters can be repeated but no two numbers can be the same?
   (c) How many different plates can be made if repetitions of numbers and letters are allowed except that no plate can have four zeros?
2. The board of a large corporation has six members willing to be nominated for office. How many different "president/vice president/treasurer" slates could be submitted to the stockholders?
3. The Alpha Beta Zeta sorority is trying to fill a pledge class of nine new members during fall rush. Among the twenty-five available candidates, fifteen have been judged marginally acceptable and ten highly desirable. How many ways can the pledge class be chosen to give a two-to-one ratio of highly desirable to marginally acceptable candidates?
4. An urn contains six chips, numbered 1 through 6. Two are chosen at random and their numbers are added together. What is the probability that the resulting sum is equal to 5?
5. A fair coin is tossed three times. Let X be the number of heads minus the number of tails. Find $p_X(k)$.
6. Two fair dice are rolled. Let X denote the larger of the two numbers showing (if the two faces are equal, X is that common value). Find $p_X(k)$.

Answer Key

1. (a) 6,760,000. (b) 3,407,040. (c) 6,759,324.
2. 120.
3. 95,550.
4. $\dfrac{2}{15}$.
5. $p_X(-3) = \frac{1}{8}, \quad p_X(-1) = \frac{3}{8}, \quad p_X(1) = \frac{3}{8}, \quad p_X(3) = \frac{1}{8}$.
6. $p_X(k) = \dfrac{2k-1}{36}$ for $k = 1, 2, \ldots, 6$; that is, $\frac{1}{36}, \frac{3}{36}, \frac{5}{36}, \frac{7}{36}, \frac{9}{36}, \frac{11}{36}$.

Challenge Problem

1. A mathematician keeps a matchbox in each of two pockets, the left and the right, each box initially holding ten matches. Whenever a match is needed, a pocket is chosen at random, each pocket being equally likely, and one match is taken from that box. Eventually the mathematician reaches into a pocket and finds that box empty. Let X be the number of matches remaining in the *other* box at that moment. Find the distribution $p_X(k)$ for $k = 0, 1, \ldots, 10$.