Lecture 01: Basic Probability Language

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1. A graduating engineer has signed up for three job interviews. She intends to categorize each one as being either a "success" or a "failure" depending on whether it leads to a plant trip. Write out the appropriate sample space. What outcomes are in the event A: Second success occurs on third interview? In B: First success never occurs?

2. During orientation week, the latest Spiderman movie was shown twice at State University. Among the entering class of 6000 freshmen, 850 went to see it the first time, 690 the second time, while 4700 failed to see it either time. How many saw it twice?

3. Let A and B be two events defined on S. If the probability that at least one of them occurs is 0.3 and the probability that A occurs but B does not occur is 0.1, what is $P(B)$?

4. In the game of "odd man out" each player tosses a fair coin. If all the coins turn up the same except for one, the player tossing the different coin is declared the odd man out and is eliminated from the contest. Suppose that three people are playing. What is the probability that someone will be eliminated on the first toss?

5. If State's football team has a 10% chance of winning Saturday's game, a 30% chance of winning two weeks from now, and a 65% chance of losing both games, what are their chances of winning exactly once?

6. A fair coin is tossed three times. What is the probability that at least two heads will occur given that at most two heads have occurred?

7. Two fair dice are rolled. What is the probability that the number on the first die was at least as large as 4 given that the sum of the two dice was 8?

8. Suppose that $P(A \cap B) = 0.2$, $P(A) = 0.6$, and $P(B) = 0.5$.
   (a) Are A and B mutually exclusive?
   (b) Are A and B independent?
   (c) Find $P(A^C \cup B^C)$.

9. A toy manufacturer buys ball bearings from three different suppliers—50% of her total order comes from supplier 1, 30% from supplier 2, and the rest from supplier 3. Past experience has shown that the quality-control standards of the three suppliers are not all the same. Two percent of the ball bearings produced by supplier 1 are defective, while suppliers 2 and 3 produce defective bearings 3% and 4% of the time, respectively. What proportion of the ball bearings in the toy manufacturer's inventory are defective?

10. Give an example of three events such that they are pairwise independent but not mutually independent.

Textbook: Larsen & Marx, *An Introduction to Mathematical Statistics and Its Applications*, Fifth Edition.

Topics: Sample spaces, events, unions, intersections, complements, Kolmogorov axioms, inclusion-exclusion (Sections 2.1–2.3). **Answer Key**

1. $S = \{\text{SSS, SSF, SFS, SFF, FSS, FSF, FFS, FFF}\}$; $A = \{\text{SFS, FSS}\}$; $B = \{\text{FFF}\}$.
2. 240 students saw the movie twice.
3. $P(B) = 0.2$.
4. $\frac{3}{4}$.
5. 0.30 (30%).
6. $\frac{3}{7}$.
7. $\frac{3}{5}$.
8. (a) No. (b) No. (c) 0.8.
9. 0.027 (2.7%).
10. Many constructions possible. Toss three coins A_{ij} = Exactly one of i or j is a head. Now consider A_{12}, A_{13}, A_{23}.

(Optional, Challenge) N people line up to board an airplane. Each has a boarding pass with assigned seat. However, the first person to board has lost his boarding pass and takes a random seat. After that, each person takes the assigned seat if it is unoccupied, and one of unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in his assigned seat for N=3? [Harder: What is it for N=100?]. No guesses are allowed. Only use axioms or claims proved in class to compute.