Internal Assessment I

Statistics for Economics 26–27

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Maximum Marks: 34 **Time:** 90 minutes

Steps carry marks, not the final answer. No marks will be given for a correct final answer without steps.

1. **(3 marks)** Let $X \sim \text{Binomial}(2, 0.2)$. Using the definition of expectation, compute $\mathbb{E}[X]$.

2. **(3 marks)** A warranty on a new delivery van covers eighteen free servicings by the dealer. Suppose the van needs servicing, on average, twice a year. What is the average time it will take for the warranty to be used up? You must set up the random variables and use linearity of expectation to solve this problem. Intuitive answers will not carry marks.

3. **(2 marks each)**

   (a) The moment-generating function of X is

   $$M_X(t) = (1-2t)^{-3}, \qquad t < \tfrac{1}{2}.$$

   Find $\mathbb{E}[X^2]$.

   (b) Two continuous random variables (X, Y) have joint density

   $$f_{X,Y}(x, y) = 6x^2y, \qquad 0 \le x \le 1,\ 0 \le y \le 1.$$

   Find the marginal density $f_Y(y)$.

   (c) For two events A and B you are told that $\mathbb{P}(A) = 0.7$ and $\mathbb{P}(B) = 0.6$. Find the largest and the smallest possible values of $\mathbb{P}(A \cap B)$.

   (d) Is the conditional expectation $\mathbb{E}[X \mid Y]$ a number or a random variable? Explain briefly.

4. **(6 marks)** In each part, identify the distribution involved and use it to compute the required probability.

   (a) A loan officer reviews applications one at a time and approves each, independently, with probability 0.3. What is the probability that her first approval is the fourth application she reviews?

   (b) Annual rainfall (in cm) at a weather station is normally distributed with mean 100 and standard deviation 20. What is the probability that a given year's rainfall exceeds 130 cm?

5. **(8 marks)** Let X and Y be two income shares of a household with joint density

$$f_{X,Y}(x,y) = c\,xy, \qquad 0 \leq x \leq y \leq 1.$$

(a) **(3 marks)** Find the value of c that makes $f_{X,Y}$ a valid joint density.

(b) **(5 marks)** Compute $\mathbb{E}\big(X \mid Y = \frac{1}{2}\big)$.

6. **(10 marks)** Passengers arrive at a bus stand at an average rate of 8 per minute; the number who have arrived t minutes after opening is distributed as Poisson$(8t)$. The bus arrives after a time that is exponentially distributed with mean 5 minutes. It has 40 seats, of which the number already occupied is equally likely to be any integer from 1 to 40. Find the expected number of passengers still waiting when the bus departs.