Internal Assessment I

Statistics for Economics 26–27

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Maximum Marks: 34 **Time:** 90 minutes

Steps carry marks, not the final answer. No marks will be given for a correct final answer without steps.

1. **(3 marks)** A fair coin is tossed three times. Let X be the number of heads minus the number of tails. Compute the pmf p_X.

2. **(3 marks)** A service contract on a new university computer system provides twenty-four free repair calls from a technician. Suppose the technician is required, on average, three times a month. What is the average time it will take for the service contract to be fulfilled? You must set up the random variables and use linearity of expectation to solve this problem. Intuitive answers will not carry marks.

3. **(2 marks each)** State the formula and compute.

   (a) The moment-generating function of X is

   $$M_X(t) = \frac{2}{2-3t}, \qquad t < \tfrac{2}{3}.$$

   Find $\mathbb{E}[X]$.

   (b) Two continuous random variables (X, Y) have joint density

   $$f_{X,Y}(x, y) = x + y, \qquad 0 \le x \le 1,\ 0 \le y \le 1.$$

   Find the marginal density $f_X(x)$.

   (c) Econo-Tire is planning an advertising campaign for a new low-cost tyre. Road tests suggest the lifetimes of these tyres are normally distributed with a mean of fifty thousand miles and a standard deviation of ten thousand miles. The marketing division wants to claim that at least nine out of ten drivers will get at least forty thousand miles on a set of Econo-Tires. Is the company justified in making that claim?

   (d) For two random variables X and Y you are told that $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 9$, and $\mathrm{Var}(X+Y) = 19$. Compute $\mathrm{Cov}(X, Y)$.

4. **(6 marks)** In each part, identify the distribution involved and use it to compute the required probability.

(a) A rural bank branch receives loan applications at an average rate of two per hour. What is the probability that exactly four applications arrive in a given hour?

(b) The duration of an unemployment spell (in weeks) is memoryless with mean 10 weeks. What is the probability that a worker's spell lasts more than fifteen weeks?

5. **(10 marks)** A household's budget is split between two categories: let X be the fraction of income spent on food and Y the fraction spent on housing. Their joint behaviour is modelled by the density

$$f_{X,Y}(x,y) = c\,x^2 y, \qquad 0 \le y \le x \le 1.$$

(a) **(3 marks)** Find the value of c that makes $f_{X,Y}$ a valid joint density.

(b) **(7 marks)** Compute $\mathbb{E}\big(X \mid Y = \frac{1}{2}\big)$.

6. **(8 marks)** A rural NGO runs a financial-literacy programme and wants to know whether it raises loan repayment. Let $D = 1$ mark a borrower who took the programme and $D = 0$ one who did not, and let $Y = 1$ if the borrower repays. Each borrower has an unobserved discipline type $A \in \{H, L\}$, with $\mathbb{P}(A = H) = \mathbb{P}(A = L) = \frac{1}{2}$. Enrolment depends on the type,

$$\mathbb{P}(D = 1 \mid A = H) = 0.6, \qquad \mathbb{P}(D = 1 \mid A = L) = 0.2,$$

and repayment depends only on type and programme. Each entry of the table below is the repayment probability $\mathbb{P}(Y = 1 \mid A, D)$ for a borrower of type A (row) and programme status D (column):

$\mathbb{P}(Y = 1 \mid A, D)$	$D = 1$	$D = 0$
$A = H$	0.75	0.60
$A = L$	0.45	0.30

(a) **(2 marks)** For each discipline type, by how much does taking the programme change that borrower's probability of repaying?

(b) **(2 marks)** Compute $\mathbb{P}(A = H \mid D = 1)$.

(c) **(4 marks)** The NGO reports $\mathbb{E}[Y \mid D = 1] - \mathbb{E}[Y \mid D = 0]$ as the effect of the programme. In one or two sentences, explain why this figure differs from the effect found in part (a), and name what drives the difference.