Week-2,Stats 25-26

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1. Larsen and Marx (5th ed) Problems 3.2.3,3.2.8,3.2.15,
2. Show that the mean of a Bernoulli(p) random variable is p. Prove that the variance is $p - p^2$.
3. Suppose that X and Y are independent binomial random variables with parameters (n, p) and (m, p). Argue probabilistically (no computations necessary) that $X + Y$ is binomial with parameters $(n + m, p)$.
4. An urn contains five red, three orange, and two blue balls. Two balls are randomly selected. What is the sample space of this experiment? Let X represent the number of orange balls selected. What are the possible values of X? Calculate the pmf of X, mean and variance of X.
5. Let (Ω, P) be a sample space and a probability measure. Let E be an event. Define the **Indicator** random variable I_E
$$I_E = \begin{cases} 1, & \text{if } E \text{ occurs} \\ 0, & \text{if } E^c \text{ occurs} \end{cases}$$
Compute the pmf, mean and variance of the random variable.
6. If X is distributed as Binomial$(79, 0.25)$, which is the most likely value of the outcome? [In other words, find k such that $P(X = k)$ is maximum.]
7. Consider three trials, each of which is either a success or not. Let X denote the number of successes. Suppose that $\mathbb{E}[X] = 1.8$.
   (a) What is the largest possible value of $P\{X = 3\}$?
   (b) What is the smallest possible value of $P\{X = 3\}$?

   (Challenge) Solve the above problem by changing the mean from 1.8 to 2.5.
8. Consider 5 independent flips of a coin having probability p of landing heads. Say a changeover occurs whenever an outcome differs from the one preceding it. For instance, if the results of the flips are $HHTHTHHT$, then there are a total of five changeovers. Let C be the number of change overs in 6 tosses.
   (a) What is the probability that there is a change over at the 4th toss?
   (b) What is the probability that there is a change over at the 4th toss given there is a change over at the 3rd toss? Are these events independent?
   (c) (Challenge) Find the range of C, the pmf and the mean.