Internals-1 Supplementary
Statistics for Economists 25-26

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Maximum Marks: 20 **Time:** 60 minutes

The exam is out of 24 marks and you can score a maximum of 20 marks.

1. (1 mark each) Answer briefly. No justification needed.
   (a) If I toss a fair coin, what is the probability of seeing heads?
   (b) If $X \sim \text{Poisson}(\lambda)$, what is $\mathbb{E}[X]$?
   (c) State the definition of covariance between X and Y.
   (d) If the probability density function of a random variable is $f_X(x) = e^{-2|x|}$, what is the probability that $X = 0$?
   (e) If $X \sim \text{Uniform}(0, 1)$ and $Y \sim \text{Uniform}(0, 1)$ are independent, what is $\Pr(X + Y \leq 1/2)$?
2. (4 marks) Consider three trials, each of which is either a success or not. Let X denote the number of successes. Suppose that $\mathbb{E}[X] = 1.5$.
   (a) What is the largest possible value of $\Pr(X = 3)$?
   (b) What is the smallest possible value of $\Pr(X = 3)$?
3. (5 marks) Two teams play a series of games; each game is independently won by team A with probability p and by team B with probability $1 - p$. The series ends as soon as one team has won 2 games.

   Let X be the number of games won by the loser when the series ends.
   (a) Construct the sample space of terminating sequences of games and give the probability of each outcome.
   (b) Define the random variable X on this sample space (explicitly map each terminating sequence to the corresponding value of X).
   (c) From this, compute the probability mass function of X and then compute $\mathbb{E}[X]$.
4. (3 marks) Let X and Y be random variables with finite expectations. Show that

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y].$$

   (You may assume the discrete case with $\Pr(\cdot)$ defined on a countable sample space.)
5. (3 marks) Let X and Y be independent $\text{Uniform}(0, 1)$ random variables. Define $Z = X + Y$.
   (a) Find $\mathbb{E}[X \mid Z = z]$ for $0 < z < 2$.

(b) Hence compute $\mathbb{E}[X \mid Z]$ as a random variable.

6. (4 marks) Let $X_1, X_2, \ldots, X_n$ be independent Exp(1) random variables (exponential with mean 1). Define $M_n = \max\{X_1, \ldots, X_n\}$.

   (a) Show that $\Pr(M_n \leq t) = (1 - e^{-t})^n$, $t \geq 0$.

   (b) Deduce that $M_n - \log n$ converges in distribution as $n \to \infty$, and identify the cdf of the limiting distribution.