Problems in school probability

Refresher

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Example 1.4 Suppose cards numbered one through ten are placed in a hat, mixed up, and then one of the cards is drawn. If we are told that the number on the drawn card is at least five, then what is the conditional probability that it is ten?

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

Example 1.8 Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men selects his own hat?

An urn contains five white chips, four black chips, and three red chips. Four chips are drawn sequentially and without replacement. What is the probability of obtaining the sequence (white, red, white, black)?

A student may be **prepared** or **unprepared** for an exam. The chance she is prepared is 0.7.

- If she is **prepared**, she answers a question correctly with probability 0.9.
- If **unprepared**, the chance is 0.2.

What is the probability she answers the question correctly?

(a) Solve using the law of total probability.

(b) Now suppose the question was answered correctly. What is the probability she was prepared?

A card is drawn at random from a standard 52-card deck. Let

- A: the card is a heart
- B: the card is a queen
- C: the card is a red card

(a) Are A and B independent?

(b) Are A and C independent?

(c) Are A and C mutually exclusive?

(d) General: Can two non-empty mutually exclusive events be independent?

2.4.35. A study has shown that seven out of ten people will say “heads” if asked to call a coin toss. Given that the coin is fair, though, a head occurs, on the average, only five times out of ten. Does it follow that you have the advantage if you let the other person call the toss? Explain.

14. The probability of winning on a single toss of the dice is p. A starts, and if he fails, he passes the dice to B, who then attempts to win on her toss. They continue tossing the dice back and forth until one of them wins. What are their respective probabilities of winning?

There are three boxes:

1. a box containing two gold coins,
2. a box containing two silver coins,
3. a box containing one gold coin and one silver coin.

A coin withdrawn at random from one of the three boxes happens to be a gold. What is the probability the other coin from the same box will also be a gold coin?

***30.** Bill and George go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with probability 0.7, whereas George, independently, hits the target with probability 0.4.

(a) Given that exactly one shot hit the target, what is the probability that it was George's shot?

(b) Given that the target is hit, what is the probability that George hit it?

Challenge:

Andy, Bob, and Charley have gotten into a disagreement over a female acquaintance, Donna, and decide to settle their dispute with a three-cornered pistol duel. Of the three, Andy is the worst shot, hitting his target only 30% of the time. Charley, a little better, is on-target 50% of the time, while Bob never misses (see Figure 2.5.3). The rules they agree to are simple: They are to fire at the targets of their choice in succession, and cyclically, in the order Andy, Bob, Charley, and so on, until only one of them is left standing. On each "turn," they get only one shot. If a combatant is hit, he no longer participates, either as a target or as a shooter.

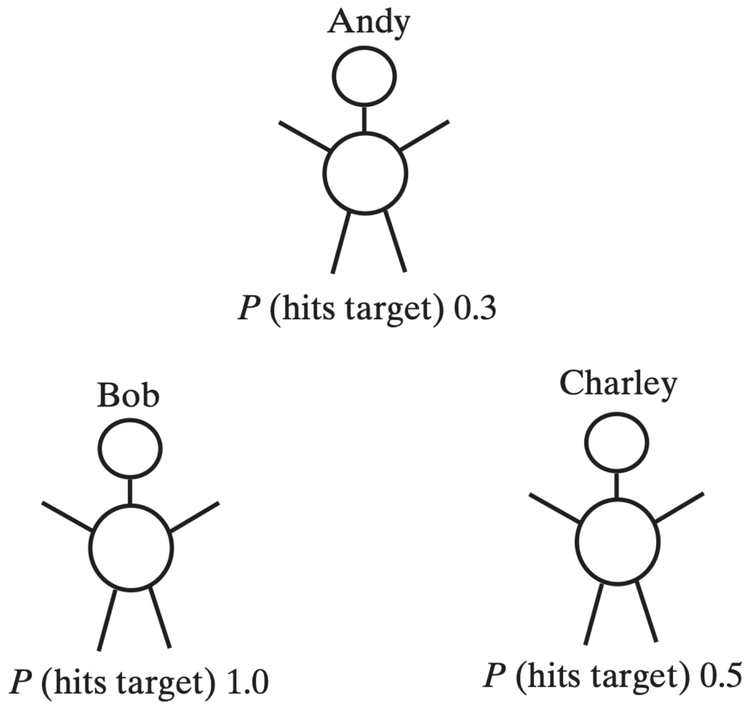

Figure 2.5.3

What should Andy do maximise his chance of survival?

Challenge:

Suppose a mathematician carries two matchboxes at all times: one in his left pocket and one in his right. Each time he needs a match, he is equally likely to take it from either pocket. Suppose he reaches into his pocket and discovers for the first time that the box picked is empty. If it is assumed that each of the matchboxes originally contained N matches, what is the probability that there are exactly k matches in the other box?