Internals-I
Statistics for Economists 25-26

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Maximum Marks: 40 **Time:** 90 minutes

The total marks is 48, you can only score a maximum of 40 marks. Choose the problems wisely. Some low markers could be hard.

1. (2 marks each) Answer the following questions objectively. No justifications necessary.

   (a) A box contains ONE blue ball. If a ball is drawn at random, what is the probability that drawn ball is blue colored?

   (b) Events A and B are independent if $P(A \cap B) = P(A)P(B)$. If $P(A) = 0.3$ and $P(A \cap B) = 0.1$, If A and B are independent, then what is $P(B)$?

   (c) Write the law of linearity of expectation.

   (d) Let $F_X(x)$ be a cdf of a random variable X, then what is the value of

   $$\lim_{x \to \infty} F_X(x)?$$

   (e) If X is uniformly distributed in the interval $[0, 1]$, what is $P(X = 0.7)$?

   (f) If $M(x)$ is the moment generating function of the normal distribution with mean μ and variance σ^2, what is the value of $M''(0) - M'(0)^2$?

   (g) Let X and Y be Bernoulli($\frac{1}{2}$) random variables. Write down a joint pmf $p_{X,Y}(x, y)$ on $\{0, 1\} \times \{0, 1\}$ such that $\text{Cov}(X, Y) = 0.2$. (The marks are for the joint pmf table only. Dont show any derivation!)

2. (5 marks) Let $Y \sim \text{Bernoulli}(p)$. Prove $\mathbb{E}[Y] = p$ and $\text{Var}(Y) = p(1 - p)$.

3. (4 marks) A firm segments its customer base into three product-preference types labelled A, B, C with counts 5, 3 and 2 respectively. Two customers are sampled without replacement. Let X be the number of sampled customers who prefer product B.

   (a) Write down the sample space (outcomes described by types) for the two draws.

   (b) Give the possible values of X.

   (c) Compute the pmf of X.

4. (4 marks) A credit-scoring model classifies loans as either high-risk or low-risk. From historical data:

   - About 2% of all loans actually default.
   - Among the loans that actually default, the model correctly labels 95% of them as high-risk.

- Among the loans that do not default, the model incorrectly labels 10% of them as high-risk.

(a) Find the probability that a randomly chosen loan is labeled high-risk.

(b) Find the probability that a randomly chosen loan actually defaults given that it is labeled high-risk.

5. (4 marks) Let X be a random variable with moment generating function

$$M_X(t) = \frac{2}{2-3t}, \qquad t < \frac{2}{3}.$$

(a) (2 marks) Find $\mathbb{E}[X]$.

(b) (2 marks) Find the value M such that $\Pr(X > M) = \frac{1}{2}$.

6. (6 marks) There are two procurement departments:

- Department A orders items labelled $1, 2, 3$ (one of these chosen uniformly when an order is fulfilled).
- Department B orders items labelled $1, 2, 3, 4, 5$ (one chosen uniformly).

A department is chosen at random (probability 1/2 each), then one item is drawn uniformly from that department's set. Let Z be the label observed.

(a) Compute $\mathbb{E}[Z]$.

(b) Compute $\mathrm{Var}(Z)$.

If you use law of total expectation and law of total variance, state the mean and variance of discrete uniform distribution and directly use it.

7. (5 marks) Suppose the joint pdf of two continuous random variables (X, Y) is

$$p_{X,Y}(x, y) = \frac{cxy}{4}, \qquad x, y \in (1, 2).$$

(a) Determine the normalising constant c.

(b) Compute the marginal pdf $p_X(x)$. Argue that p_Y must be the same as p_X without computation.

(c) Are X and Y independent? Justify briefly.

8. (6 marks) For each $k = 1, 2, \ldots, n$ let

$$X_k \sim \mathrm{Bernoulli}\Big(\frac{1}{k+1}\Big).$$

Assume $X_1, X_2, \cdots, X_n$. Define $M_n = \max\{X_1, \ldots, X_n\}$ (so $M_n = 1$ iff at least one success occurs).

(a) Show that $\Pr(M_n = 0) = \dfrac{1}{n+1}$ and simplify this product.

(b) Deduce the exact distribution of M_n and compute $\mathbb{E}[M_n]$.

(c) Compute

$$\lim_{n\to\infty} \mathbb{E}[M_n]$$

and give a one-sentence interpretation.