MADRAS SCHOOL OF ECONOMICS

UNDERGRADUATE PROGRAMME IN ECONOMICS (HONOURS) [2024-27]

SEMESTER 3 [JULY – NOVEMBER, 2025]

REGULAR END TERM EXAMINATION, NOVEMBER 2025

Course Name: Statistics for Economics, **Course Code:** CC08

Duration: 2 Hours **Maximum Marks: 60**

Instructions: *For part A write short answers (most preferably quote a theorem/identity or justify in a sentence or two). For part B, writing relevant formulae and quoting correct definitions helps me give you part marks.*

Part A: Answer all questions (1 mark x 10 questions = 10 marks)

1. Why does the sample variance for n data points have a $n-1$ in the denominator?
2. Can the heights of the male population of Chennai be distributed as a standard normal? Explain.
3. Explain the precise meaning of the term "almost surely" in statistics and probability theory.
4. A reporter surveyed a large number of prison inmates and found that a majority were first-born children. This led to discussion on why first-borns are more likely to be criminals, citing parenting styles and societal pressures. As a statistician, what do you suspect might be the reason?
5. What is the distribution of $\sum_{i=1}^{n} \frac{(Y_i - \mu)^2}{\sigma^2}$ if $Y_1, Y_2, \ldots, Y_n$ are drawn from i.i.d Normal(μ, σ^2)?
6. Prove: If W is an unbiased estimator for θ and W is non-constant almost surely, then W^2 is not an unbiased estimator for θ^2.
7. Compute the Cramer-Rao lower bound (the right hand side of the inequality) for the exponential distribution.
8. Write down the probability density function for a bivariate normal (X,Y) with mean (μ_X, μ_Y) and covariance matrix Σ.
9. A random sample of twenty cell phones yields absorption rates (in watts per kilogram) for radio frequency energy. The FCC safety limit is 1.6 W/kg. The sample mean and standard deviation of the absorption rates are $\bar{x} = 1.211$ W/kg and $s = 0.5$ W/kg, respectively. Using a 95% confidence interval for the true mean absorption rate and assuming a t-statistic is used, conclude whether the phones are safe to use.
10. State the general formula for the probability density function of the k-th order statistic $X_{(k)}$ from a sample of n i.i.d. continuous random variables with pdf f and CDF F.

Part B: Answer any five. (5 mark x 10 questions = 50 marks)

11. State and prove Rao-Blackwell theorem. Explain how it is useful and illustrate it using an example. Work out the example clearly and rigorously.

12. Answer the following questions:

    (a) [3 marks] An herbalist believes a new supplement affects the IQ scores of children with mild attention deficit disorder (ADD). For a random sample of 10 such children, the sample mean score $\bar{y}$ is observed. The population is known to have mean $\mu = 95$ and $\sigma = 15$. If the test is conducted at significance level $\alpha = 0.1$, determine the range of $\bar{y}$ values for which H_0 would be rejected (two-sided test). Show the derivation of your critical region.

    (b) [4 marks] A company produces a product that yields a profit of m dollars per sold unit and a loss of n dollars per unsold unit. Suppose demand V follows an exponential distribution with pdf

    $$f_V(v) = \frac{1}{\lambda} e^{-v/\lambda}, \quad v > 0.$$

    Derive the expression for the expected profit as a function of the number of items produced, and determine the optimal production quantity that maximizes expected profit.

    (c) [3 marks] Suppose that the conditional and marginal densities are given by

    $$f_{Y|X}(y|x) = \frac{2y + 4x}{1 + 4x} \quad \text{and} \quad f_X(x) = \frac{1}{3}(1 + 4x),$$

    for $0 < x < 1$ and $0 < y < 1$. Find the conditional expectation $\mathbb{E}(X|Y)$, clearly showing all steps of your derivation and verifying that the resulting density integrates to one.

13. Answer the following questions:

    (a) [3 marks] Three people, A, B, C, stand in a circle and toss a ball sequentially to the next person: $A \to B \to C \to A \ldots$ Each time the ball is tossed, the recipient may drop it with probability p, independently of all other throws. Drops do not affect turn order or stop the game (the ball is immediately recovered and the next scheduled passer throws). The game lasts for exactly 3 tosses.

        i. Write down an appropriate sample space for this experiment.

        ii. Define a random variable X that represents the number of ball drops in the 3 tosses, and show explicitly that X is a function on the sample space.

        iii. Find the probability mass function (pmf) of X.

    (b) [3 marks] Derive the MGF of square of standard normal starting from the definition of MGF and density of standard normal. You cannot quote any direct formula for the MGF of a known distribution. You may use the integral formula

    $$\int_{-\infty}^{\infty} e^{-\alpha z^2}\, dz = \sqrt{\frac{\pi}{\alpha}}, \quad \alpha > 0.$$

    (c) [4 marks] In the past, defendants convicted of grand theft auto served Y years in prison, where the pdf describing the variation in Y had the form

    $$f_Y(y) = \frac{y}{8}, \quad 0 < y \leq 4.$$

    Recent judicial reforms, though, may have impacted the punishment meted out for this particular crime. A review of 50 individuals convicted of grand theft auto five years ago showed that 6 served less than one year in jail, 12 served between one and two years, 15 served between two and three years and 17 served between three and four years. Are these data consistent with $f_Y(y)$? Perform an appropriate hypothesis test using the $\alpha = 0.05$ level of significance.

14. Answer the following questions:

(a) [5 marks] An MSE statistics student is playing an online board game with a friend who claims to be rolling a pair of fair dice each turn. The student suspects the friend is instead picking a number uniformly at random from $\{2, 3, \ldots, 12\}$. So she decides to base her conclusion on only the first total S announced by her friend. Let

$$H_0: \; S \text{ has the distribution of the sum of two fair dice,}$$
$$H_1: \; S \text{ is uniformly distributed on } \{2, \ldots, 12\}.$$

i. (3 marks) Among all tests that use this single observation and have size $\alpha = 5.56\%$, construct the test with the highest possible power.

ii. (2 marks) State its rejection region and the power under H_1.

(b) [2 marks] Let X_1 and X_2 be identically distributed Bernoulli random variables taking values in $\{0, 1\}$. Let $S = X_1 + X_2$. Suppose $S \sim \text{Binomial}(2, p)$. Show that this implies X_1 and X_2 are independent and identically distributed as Bernoulli(p).

(c) [3 marks] An engineer models the proportion Y of a task completed using the probability density function

$$f_Y(y; \theta) = \theta \, y^{\theta-1}, \qquad 0 \leq y \leq 1.$$

The engineer has a vague memory that the parameter θ is one of the three possible values in the set $\{1, 3, 6\}$. In a previous project, the observed completion proportions for five tasks were $0.77, \; 0.82, \; 0.92, \; 0.94, \; 0.98$. Find the maximum likelihood estimate of θ based on these data.

15. Answer the following questions:

(a) [2 marks] The Pew Research Center did a survey of 2253 adults and discovered that 63% of them had broadband Internet connections in their homes. The survey report noted that this figure represented a "significant jump" from the similar figure of 54% from two years earlier. One way to define "significant jump" is to show that the earlier number does not lie in the 95% confidence interval. Was the increase significant by this definition?

(b) [3 marks] Show that if X, Y are independent and identically distributed as $\mathcal{N}(0, \sigma^2)$, then

$$\frac{X + Y}{|X - Y|}$$

is t-distributed. [Hint: Recall t-statistic?]

(c) [5 marks] Two analysts at an insurance company disagree about the mean number of daily claims. Anyaay believes it remains at 10 claims per day, while Bhayaanak argues that it has increased to 11. Assume that the number of claims on each of n independent days follows a Poisson distribution with mean λ. They agree to perform the most powerful level-5% test of

$$H_0: \lambda = 10 \quad \text{vs} \quad H_1: \lambda = 11,$$

and that n is large enough for a Central Limit Theorem approximation (with a one-sided continuity correction) to be used if needed.

i. [3 marks] Derive the form of the most powerful test at the 5% level, and using an appropriate CLT approximation, obtain the critical region in terms of n for this test. Then express the approximate power under H_1 as a function of n.

ii. [2 marks] Hence determine approximately how many days of claim data are required so that the level-5% test has 95% power to detect an increase from 10 to 11 claims per day. Show your reasoning and state the final value of n.

16. Answer the following questions:

   (a) [5 marks] Let $X_1, \dots, X_n$ be i.i.d. geometric random variables with pmf

$$\Pr(X_i = k \mid \theta) = (1-\theta)^{k-1}\theta, \quad k = 1, 2, \dots, \quad 0 < \theta < 1,$$

   and prior $\theta \sim \text{Beta}(r, s)$. Let $T_n = \sum_{i=1}^{n} X_i$.

      i. [1 marks] Derive the posterior distribution $\pi(\theta \mid X_{1:n})$ and the posterior mean $\hat{\theta}_n(X_{1:n})$ in terms of r, s, n, and T_n.

      ii. [4 marks] Switching to a frequentist view, and treating θ as a fixed but unknown constant, determine whether the statistic $\hat{\theta}_n$ is an unbiased estimator of θ for the geometric pmf. Is it consistent?

   (b) [5 marks] Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables from the uniform distribution

$$f(x;\theta) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta, \\ 0, & \text{otherwise,} \end{cases}$$

   where $\theta > 0$ is unknown.

      i. (1 mark) State the maximum likelihood estimator (MLE) of θ.

      ii. (4 marks) Prove that the MLE is a consistent estimator of θ.

17. Answer the following questions:

   (a) [4 marks] Let X_1, X_2, and X_3 be independent Bernoulli random variables with common but unknown parameter $p = P(X_i = 1)$. It is known that $S = X_1 + X_2 + X_3$ is a sufficient statistic for p. Show that the linear combination $T = X_1 + 2X_2 + 3X_3$ is not sufficient for p.

   (b) [6 marks] Suppose we have a possibly biased coin with probability of Head p, and an urn containing w>0 white balls and b>0 black balls. A single trial is conducted as follows:

   Toss the coin once. If the outcome is Head, add one white ball to the urn. If the outcome is Tail, add one black ball to the urn. Then draw one ball at random from the urn, which now contains $w + b + 1$ balls, and record its colour and place it back into the urn.

   Let X denote the number of times a white ball is recorded in n independent repeated trials of this experiment.

      i. (2 marks) Show that the probability of recording a white ball in a single trial is

$$q(p) = \frac{w + p}{w + b + 1}.$$

      ii. (2 marks) Write down the likelihood function for p.

      iii. (2 marks) For $w = 1, b = 2, n = 5$ let the maximum likelihood estimator of p be $\hat{p}$, is there a value of X for which $0 < \hat{p} < 1$.

Common Probability Distributions

Notation. Unless otherwise stated:

$$\mathbb{E}[X] \text{ denotes the mean,} \quad \mathrm{Var}(X) \text{ the variance,} \quad f(x) \text{ the pdf or pmf,}$$

$$F(x) \text{ the cdf,} \quad M_X(t) \text{ the mgf.}$$

$\Gamma(\cdot)$ is the gamma function, $B(\cdot,\cdot)$ the beta function, and $\Phi(\cdot)$ the standard normal cdf. Floor $\lfloor\cdot\rfloor$ and ceiling $\lceil\cdot\rceil$ indicate integer parts.

Bernoulli(p)

$$x \in \{0,1\}, \quad f(x) = p^x(1-p)^{1-x}, \quad M_X(t) = 1 - p + pe^t.$$

$$\mathbb{E}[X] = p, \quad \mathrm{Var}(X) = p(1-p), \quad \text{Mode: } 1 \text{ if } p > 1/2, \text{ else } 0.$$

Binomial(n, p)

$$x = 0, 1, \ldots, n, \quad f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad M_X(t) = (1 - p + pe^t)^n.$$

$$\mathbb{E}[X] = np, \quad \mathrm{Var}(X) = np(1-p), \quad \text{Mode: } \lfloor (n+1)p \rfloor.$$

Geometric(p) (number of trials until first success)

$$x = 1, 2, \ldots, \quad f(x) = p(1-p)^{x-1}, \quad M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \ t < -\ln(1-p).$$

$$\mathbb{E}[X] = \frac{1}{p}, \quad \mathrm{Var}(X) = \frac{1-p}{p^2}, \quad \text{Mode: } 1.$$

Poisson(λ)

$$x = 0, 1, 2, \ldots, \quad f(x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad M_X(t) = \exp[\lambda(e^t - 1)].$$

$$\mathbb{E}[X] = \lambda, \quad \mathrm{Var}(X) = \lambda, \quad \text{Mode: } \lfloor \lambda \rfloor.$$

Hypergeometric(N, K, n)

$$x = \max(0, n - N + K), \ldots, \min(n, K), \quad f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}.$$

$$\mathbb{E}[X] = n\frac{K}{N}, \quad \mathrm{Var}(X) = n\frac{K}{N}\left(1 - \frac{K}{N}\right)\frac{N-n}{N-1}.$$

Uniform(a, b)

$$a \le x \le b, \quad f(x) = \frac{1}{b-a}, \quad M_X(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}.$$

$$\mathbb{E}[X] = \frac{a+b}{2}, \quad \mathrm{Var}(X) = \frac{(b-a)^2}{12}.$$

Exponential(λ)

$$x \ge 0, \quad f(x) = \lambda e^{-\lambda x}, \quad F(x) = 1 - e^{-\lambda x}, \quad M_X(t) = \frac{\lambda}{\lambda - t}, \ t < \lambda.$$

$$\mathbb{E}[X] = \frac{1}{\lambda}, \quad \mathrm{Var}(X) = \frac{1}{\lambda^2}, \quad \text{Mode: } 0.$$

Gamma(α, λ)

$$x > 0, \quad f(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \quad M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha}.$$

$$\mathbb{E}[X] = \frac{\alpha}{\lambda}, \quad \mathrm{Var}(X) = \frac{\alpha}{\lambda^2}, \quad \text{Mode: } \frac{\alpha - 1}{\lambda} \ (\alpha > 1).$$

Normal(μ, σ^2)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \quad M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}.$$

$$\mathbb{E}[X] = \mu, \quad \mathrm{Var}(X) = \sigma^2, \quad \text{Mode, Median: } \mu.$$

Chi-square(k)

$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}, \quad M_X(t) = (1-2t)^{-k/2}.$$

$$\mathbb{E}[X] = k, \quad \mathrm{Var}(X) = 2k, \quad \text{Mode: } k - 2 \ (k > 2).$$

Student-$t(\nu)$

$$f(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\,\Gamma(\nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}.$$

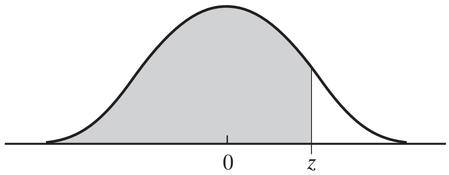
$$\mathbb{E}[X] = 0 \ (\nu > 1), \quad \mathrm{Var}(X) = \frac{\nu}{\nu - 2} \ (\nu > 2), \quad \text{Mode: } 0.$$

Beta(α, β)

$$0 < x < 1, \quad f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)},$$

$$\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta}, \quad \mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}, \quad \text{Mode: } \frac{\alpha-1}{\alpha+\beta-2} \ (\alpha, \beta > 1).$$

Table A.1 Cumulative Areas under the Standard Normal Distribution



z	0	1	2	3	4	5	6	7	8	9
−3.	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0300	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0570	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

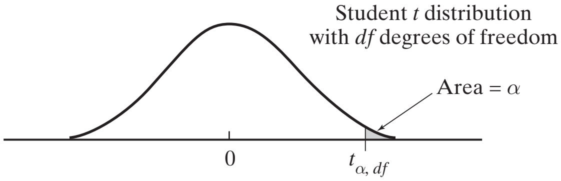
(*cont.*)

Table A.1 Cumulative Areas under the Standard Normal Distribution (*cont.*)

z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Source: From Samuels/Witmer, *Statistics for Life Sciences*, Table 3, p. 675, © 2003 Pearson Education, Inc. Reproduced by permission of Pearson Education, Inc.

Table A.2 Upper Percentiles of Student *t* Distributions



	α						
df	0.20	0.15	0.10	0.05	0.025	0.01	0.005
1	1.376	1.963	3.078	6.3138	12.706	31.821	63.657
2	1.061	1.386	1.886	2.9200	4.3027	6.965	9.9248
3	0.978	1.250	1.638	2.3534	3.1825	4.541	5.8409
4	0.941	1.190	1.533	2.1318	2.7764	3.747	4.6041
5	0.920	1.156	1.476	2.0150	2.5706	3.365	4.0321
6	0.906	1.134	1.440	1.9432	2.4469	3.143	3.7074
7	0.896	1.119	1.415	1.8946	2.3646	2.998	3.4995
8	0.889	1.108	1.397	1.8595	2.3060	2.896	3.3554
9	0.883	1.100	1.383	1.8331	2.2622	2.821	3.2498
10	0.879	1.093	1.372	1.8125	2.2281	2.764	3.1693
11	0.876	1.088	1.363	1.7959	2.2010	2.718	3.1058
12	0.873	1.083	1.356	1.7823	2.1788	2.681	3.0545
13	0.870	1.079	1.350	1.7709	2.1604	2.650	3.0123
14	0.868	1.076	1.345	1.7613	2.1448	2.624	2.9768
15	0.866	1.074	1.341	1.7530	2.1315	2.602	2.9467
16	0.865	1.071	1.337	1.7459	2.1199	2.583	2.9208
17	0.863	1.069	1.333	1.7396	2.1098	2.567	2.8982
18	0.862	1.067	1.330	1.7341	2.1009	2.552	2.8784
19	0.861	1.066	1.328	1.7291	2.0930	2.539	2.8609
20	0.860	1.064	1.325	1.7247	2.0860	2.528	2.8453
21	0.859	1.063	1.323	1.7207	2.0796	2.518	2.8314
22	0.858	1.061	1.321	1.7171	2.0739	2.508	2.8188
23	0.858	1.060	1.319	1.7139	2.0687	2.500	2.8073
24	0.857	1.059	1.318	1.7109	2.0639	2.492	2.7969
25	0.856	1.058	1.316	1.7081	2.0595	2.485	2.7874
26	0.856	1.058	1.315	1.7056	2.0555	2.479	2.7787
27	0.855	1.057	1.314	1.7033	2.0518	2.473	2.7707
28	0.855	1.056	1.313	1.7011	2.0484	2.467	2.7633
29	0.854	1.055	1.311	1.6991	2.0452	2.462	2.7564
30	0.854	1.055	1.310	1.6973	2.0423	2.457	2.7500
31	0.8535	1.0541	1.3095	1.6955	2.0395	2.453	2.7441
32	0.8531	1.0536	1.3086	1.6939	2.0370	2.449	2.7385
33	0.8527	1.0531	1.3078	1.6924	2.0345	2.445	2.7333
34	0.8524	1.0526	1.3070	1.6909	2.0323	2.441	2.7284

(*cont.*)

Table A.2 Upper Percentiles of Student t Distributions (*cont.*)

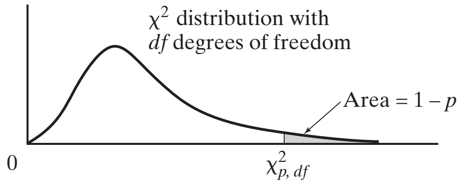
	α						
df	0.20	0.15	0.10	0.05	0.025	0.01	0.005
35	0.8521	1.0521	1.3062	1.6896	2.0301	2.438	2.7239
36	0.8518	1.0516	1.3055	1.6883	2.0281	2.434	2.7195
37	0.8515	1.0512	1.3049	1.6871	2.0262	2.431	2.7155
38	0.8512	1.0508	1.3042	1.6860	2.0244	2.428	2.7116
39	0.8510	1.0504	1.3037	1.6849	2.0227	2.426	2.7079
40	0.8507	1.0501	1.3031	1.6839	2.0211	2.423	2.7045
41	0.8505	1.0498	1.3026	1.6829	2.0196	2.421	2.7012
42	0.8503	1.0494	1.3020	1.6820	2.0181	2.418	2.6981
43	0.8501	1.0491	1.3016	1.6811	2.0167	2.416	2.6952
44	0.8499	1.0488	1.3011	1.6802	2.0154	2.414	2.6923
45	0.8497	1.0485	1.3007	1.6794	2.0141	2.412	2.6896
46	0.8495	1.0483	1.3002	1.6787	2.0129	2.410	2.6870
47	0.8494	1.0480	1.2998	1.6779	2.0118	2.408	2.6846
48	0.8492	1.0478	1.2994	1.6772	2.0106	2.406	2.6822
49	0.8490	1.0476	1.2991	1.6766	2.0096	2.405	2.6800
50	0.8489	1.0473	1.2987	1.6759	2.0086	2.403	2.6778
51	0.8448	1.0471	1.2984	1.6753	2.0077	2.402	2.6758
52	0.8486	1.0469	1.2981	1.6747	2.0067	2.400	2.6738
53	0.8485	1.0467	1.2978	1.6742	2.0058	2.399	2.6719
54	0.8484	1.0465	1.2975	1.6736	2.0049	2.397	2.6700
55	0.8483	1.0463	1.2972	1.6731	2.0041	2.396	2.6683
56	0.8481	1.0461	1.2969	1.6725	2.0033	2.395	2.6666
57	0.8480	1.0460	1.2967	1.6721	2.0025	2.393	2.6650
58	0.8479	1.0458	1.2964	1.6716	2.0017	2.392	2.6633
59	0.8478	1.0457	1.2962	1.6712	2.0010	2.391	2.6618
60	0.8477	1.0455	1.2959	1.6707	2.0003	2.390	2.6603
61	0.8476	1.0454	1.2957	1.6703	1.9997	2.389	2.6590
62	0.8475	1.0452	1.2954	1.6698	1.9990	2.388	2.6576
63	0.8474	1.0451	1.2952	1.6694	1.9984	2.387	2.6563
64	0.8473	1.0449	1.2950	1.6690	1.9977	2.386	2.6549
65	0.8472	1.0448	1.2948	1.6687	1.9972	2.385	2.6537
66	0.8471	1.0447	1.2945	1.6683	1.9966	2.384	2.6525
67	0.8471	1.0446	1.2944	1.6680	1.9961	2.383	2.6513
68	0.8470	1.0444	1.2942	1.6676	1.9955	2.382	2.6501
69	0.8469	1.0443	1.2940	1.6673	1.9950	2.381	2.6491
70	0.8468	1.0442	1.2938	1.6669	1.9945	2.381	2.6480
71	0.8468	1.0441	1.2936	1.6666	1.9940	2.380	2.6470
72	0.8467	1.0440	1.2934	1.6663	1.9935	2.379	2.6459
73	0.8466	1.0439	1.2933	1.6660	1.9931	2.378	2.6450
74	0.8465	1.0438	1.2931	1.6657	1.9926	2.378	2.6640
75	0.8465	1.0437	1.2930	1.6655	1.9922	2.377	2.6431
76	0.8464	1.0436	1.2928	1.6652	1.9917	2.376	2.6421
77	0.8464	1.0435	1.2927	1.6649	1.9913	2.376	2.6413
78	0.8463	1.0434	1.2925	1.6646	1.9909	2.375	2.6406
79	0.8463	1.0433	1.2924	1.6644	1.9905	2.374	2.6396

Table A.2 Upper Percentiles of Student *t* Distributions (*cont.*)

	α						
df	0.20	0.15	0.10	0.05	0.025	0.01	0.005
80	0.8462	1.0432	1.2922	1.6641	1.9901	2.374	2.6388
81	0.8461	1.0431	1.2921	1.6639	1.9897	2.373	2.6380
82	0.8460	1.0430	1.2920	1.6637	1.9893	2.372	2.6372
83	0.8460	1.0430	1.2919	1.6635	1.9890	2.372	2.6365
84	0.8459	1.0429	1.2917	1.6632	1.9886	2.371	2.6357
85	0.8459	1.0428	1.2916	1.6630	1.9883	2.371	2.6350
86	0.8458	1.0427	1.2915	1.6628	1.9880	2.370	2.6343
87	0.8458	1.0427	1.2914	1.6626	1.9877	2.370	2.6336
88	0.8457	1.0426	1.2913	1.6624	1.9873	2.369	2.6329
89	0.8457	1.0426	1.2912	1.6622	1.9870	2.369	2.6323
90	0.8457	1.0425	1.2910	1.6620	1.9867	2.368	2.6316
91	0.8457	1.0424	1.2909	1.6618	1.9864	2.368	2.6310
92	0.8456	1.0423	1.2908	1.6616	1.9861	2.367	2.6303
93	0.8456	1.0423	1.2907	1.6614	1.9859	2.367	2.6298
94	0.8455	1.0422	1.2906	1.6612	1.9856	2.366	2.6292
95	0.8455	1.0422	1.2905	1.6611	1.9853	2.366	2.6286
96	0.8454	1.0421	1.2904	1.6609	1.9850	2.366	2.6280
97	0.8454	1.0421	1.2904	1.6608	1.9848	2.365	2.6275
98	0.8453	1.0420	1.2903	1.6606	1.9845	2.365	2.6270
99	0.8453	1.0419	1.2902	1.6604	1.9843	2.364	2.6265
100	0.8452	1.0418	1.2901	1.6602	1.9840	2.364	2.6260
∞	0.84	1.04	1.28	1.64	1.96	2.33	2.58

Source: *Scientific Tables*, 6th ed. (Basel, Switzerland: J.R. Geigy, 1962), pp. 32–33.

Table A.3 Upper and Lower Percentiles of χ^2 Distributions

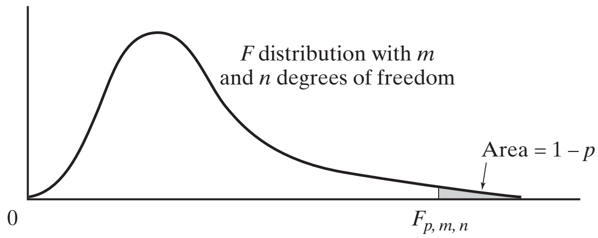


	p							
df	0.010	0.025	0.050	0.10	0.90	0.95	0.975	0.99
1	0.000157	0.000982	0.00393	0.0158	2.706	3.841	5.024	6.635
2	0.0201	0.0506	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.336	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.688	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892
31	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191
32	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486
33	17.073	19.047	20.867	23.110	43.745	47.400	50.725	54.776
34	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061

Table A.3 Upper and Lower Percentiles of χ^2 Distributions (*cont.*)

	p							
df	0.010	0.025	0.050	0.10	0.90	0.95	0.975	0.99
35	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342
36	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619
37	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892
38	20.691	22.878	24.884	27.343	49.513	53.384	56.895	61.162
39	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691
41	22.906	25.215	27.326	29.907	52.949	56.942	60.561	64.950
42	23.650	25.999	28.144	30.765	54.090	58.124	61.777	66.206
43	24.398	26.785	28.965	31.625	55.230	59.304	62.990	67.459
44	25.148	27.575	29.787	32.487	56.369	60.481	64.201	68.709
45	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957
46	26.657	29.160	31.439	34.215	58.641	62.830	66.617	71.201
47	27.416	29.956	32.268	35.081	59.774	64.001	67.821	72.443
48	28.177	30.755	33.098	35.949	60.907	65.171	69.023	73.683
49	28.941	31.555	33.930	36.818	62.038	66.339	70.222	74.919
50	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154

Source: *Scientific Tables*, 6th ed. (Basel, Switzerland: J.R. Geigy, 1962), p. 36.



The figure above illustrates the percentiles of the F distributions shown in Table A.4. Table A.4 is used with permission from Wilfrid J. Dixon and Frank J. Massey, Jr., *Introduction to Statistical Analysis* 2nd ed. (New York: McGraw-Hill, 1957), pp. 389–404.