Week 8 H.W.

BA2023, Statistics, MSE

1. Let $M(t)$ be the moment generating function of a random variable. What is the value of $\lim_{t \to -\infty} M(t)$?

2. Suppose a normal random variable N has mean 2 and variance 4, by looking up z-tables, find the first quartile of this distribution of N.

3. Let U be uniformly distributed over the interval $[0, \frac{\pi}{2}]$. Let $V = \sin U$, work out the following:

   (a) What is the probability density function of V? [Recall that you have to start from cdf of V.]

   (b) What is the mean of V? Do it directly by computing $\mathbb{E}(\sin U)$ and also do it using part 1 by computing $\mathbb{E}(V)$.

   (c) Find the median of V. Does V have a mode?

4. If X, Y are independent random variables and $Var(X + 2Y) = 37$, $Var(3X + Y) = 13$. Find $Var(X), Var(Y)$.

5. Write down the distribution of Chi-squared random variable with d degrees of freedom. What is its relation with Gamma distribution? Compute the mode and the mean of this distribution.

6. For all real x, let
$$F(x) = \frac{1}{2} + \frac{x}{2\sqrt{2}\sqrt{1 + \frac{x^2}{2}}}.$$
   Prove or disprove: F satisfies the three defining properties of the c.d.f.