Week 5 H.W.

BA2023, Statistics, MSE

1. The Gamma function is defined as
$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx.$$
The function
$$f(x) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\Gamma(k)\theta^k} \text{ for } x > 0$$
and zero else where is the p.d.f of the Gamma distribution with shape parameter $k > 0$ and scale parameter $\theta > 0$ .We will say X is distributed as Gamma(k, θ)
   (a) Check that the function is a pdf.
   (b) Fix k and plot the function for varying values of θ. What do you see?
   (c) If Y is distributed as Gamma$(k, 1)$, show that
   $$\Pr\{a < X < b\} = \Pr\{a < \theta Y < b\}.$$
   [This motivates the term scale parameter for θ.]
   (d) Gamma$(1, \theta)$ is a well known distribution. What is it?

2. Compute the mean, the variance and the MGF of the following distributions:
   (a) Binomial(n, p)
   (b) Geometric(p)
   (c) Exponential(λ)
   (d) Normal(μ, σ^2)
   (e) [Computation Optional] Gamma(k, θ)

3. Let $M_X(t)$ be a moment generating function of a random variable X. Then if X, Y are independent random variables, show that
$$M_{X+Y}(t) = M_X(t) M_Y(t).$$
   (a) Use this property to get the MGF of binomial distribution from Bernoulli.
   (b) Let n be a natural number. Comparing the MGFs of Gamma(n, λ) and Exponential(λ), what can you conclude?
   (c) Show that sum of independent and identically distributed normal random variables is a normal random variables.

4. Larsen and Marx: 3.12.1, 3.12.5,3.12.8