Week 4 H.W.

BA2023, Statistics, MSE

1. Consider the following game. A fair coin is flipped until the first tail appears; we win 2 rupees if it appears on the first toss, 4 rupees if it appears on the second toss, and, in general, 2^k rupees if it first occurs on the kth toss. Let the random variable X denote our winnings. How much should we have to pay in order for this to be a fair game? [Note: A fair game is one where the difference between the amount paid to play the game and $\mathbb{E}(X)$ is 0.]

2. A box contains r red balls and w blue balls.Picking a ball at random is called a trial. Suppose k trials are performed and balls are picked *without* replacement. Answer the following questions

   (a) If $r = w = 4$ and $k = 2$, What is the probability that you pick exactly 1 red ball?

   (b) For general r, w, k, show that the probability of picking exactly m red balls is
   $$\frac{\binom{r}{m}\binom{w}{k-m}}{\binom{r+w}{k}}$$

   (c) Let X_i denote the number of red balls picked in the ith trial. Show that $X_1, X_2, X_3, \ldots, X_k$ are Bernoulli(p) random variables. What is the value of p in terms of r and w?

   (d) Compute $\Pr\{X_1 = 1, X_2 = 1\}$. Are X_1, X_2 independent random variables?

   (e) Compute $\mathbb{E}(X_1 X_2)$.

   (f) Let $S = X_1 + X_2 + \ldots + X_k$, what is the p.m.f of S?

   (g) Calculate the mean and variance of S.

3. Let X, Y be random variables with $Y = aX + b$ where a, b are constants, show that
   $$Var(Y) = a^2 Var(X).$$
   Use this to solve Larsen and Marx Problem 3.6.16.

4. Larsen and Marx: 3.5.8,3.5.14,3.6.6,3.6.8,3.6.9, 3.6.12