Week 2 H.W.

BA2023, Mathematical Statistics, MSE

1. Look up the definition of mutual independence of events and pairwise independence of events. Can you give an example of three events that are pairwise independent but not mutually independent?

2. Company A supplies 40% of the computers sold and is late 5% of the time. Company B supplies 30% of the computers sold and is late 3% of the time. Company C supplies another 30% and is late 2.5% of the time. A computer arrives late - what is the probability that it came from Company A?

3. You are tested for a medical condition that occurs in 1 in 1000 people You are told that the test does not produce false negatives, and only produces false positives 5% of the time You test positive – what is the probability you have the condition?

4. There are three boxes, one of them has two silver coins, another has two gold coins and the third has one gold and one silver coin. A box is chosen at random and the randomly drawn coin is gold.What is the probability that the other coin in the chosen box is also gold?

   Sameer reasons as follows:"Once we see the drawn coin is gold, then the box with two silver coins is eliminated. So the chosen box can be one of the remaining two boxes. Therefore the probability is 1/2." Do you agree with Sameer?

5. Suppose you play a round of 5 games against your friend. The probability that you win any particular game is 60%. Suppose the chances of winning in different rounds are independent of each other.Let X denote the total number of wins in a round of 5 games. Write down the pmf of X and compute the expectation of X.

6. You and your friend roll a dice alternately. You start the roll. The die is rolled until a person gets 6 for the first time. Let N denote the total number of times the dice is rolled. Let A be the event that you win. Compute the following:

   (a) Range and pmf of N.Compute $\mathbb{E}(N), \mathbb{E}(N^2)$.

   (b) Define $p_{N|A}(k) := \Pr\{N = k \mid A\}$. Note that this expression is a conditional probability, conditioned on A. Compute the 'conditional probability mass function' $p_{N|A}$. Compute $\mathbb{E}(N \mid A)$,$\mathbb{E}(N^2 \mid A)$.

7. Let λ be a positive real number. Let N be a random variable, with whole numbers as range, distributed according to the p.m.f

$$p_N(k) = \frac{e^{-\lambda}\lambda^k}{k!} \text{ for k} \in \{0, 1, 2, 3, \dots\}$$

   (a) Check that it is a p.m.f.

   (b) Compute the expectations $\mathbb{E}(N)$, $\mathbb{E}(N^2)$.

Additional Practice problems (conditional probability)

Larsen and Marx 5th edition: 2.4.10,2.4.11,2.4.24,2.4.31,2.4.47,2.5.7,2.5.20.