

Week 10 H.W.

BA2023, Statistics, MSE

1. Larsen and Marx 5.2.16, 5.2.20, 5.4.7, 5.4.9, 5.4.11.
2. Define the bias of an estimator $\hat{\theta}$ as $\text{Bias}(\hat{\theta}) := \mathbb{E}(\hat{\theta}) - \theta$, and the mean squared error of the estimator as

$$MSE(\hat{\theta}) := \mathbb{E}(\hat{\theta} - \theta)^2.$$

- (a) Supposing we have data drawn from a Normal distribution. Which one has lower mean squared error (MSE) among $\hat{\sigma}_{MLE}^2$ and the sample variance?
- (b) Let $X_1, \dots, X_n$ be i.i.d. $\text{Ber}(p)$. Then what is the mean squared error of the MLE estimate of p ?
- (c) Prove

$$MSE(\hat{\theta}) = \text{Var}(\hat{\theta}) + \text{Bias}(\hat{\theta})^2.$$

3. Let $X_1, \dots, X_n$ be an i.i.d. sample from the uniform distribution on $(-a, a)$.
 - (a) Find the method of moments estimator $\hat{a}_1$. Is it unbiased?
 - (b) Find the MLE $\hat{a}_2$. Is it unbiased?
 - (c) Which estimator among the above two has lower MSE?