CC 08:Statistics for Economics

First Internals

Guidelines

- The paper is out of **40 marks** and you have **two hours**. You can use a calculator.
- You can bring **only one A4 sized cheat sheet** with your scribbles and you can write on both sides of it.
- Everyone has to attempt all questions of Section 1.
- Among Section 2 and Section 3, you can answer all the questions of **only one** of the sections. In spite of saying this, if a student answers questions from both sections, I will correct Section 3 problems only.
- You may use any theorem proved in class or any formula derived in class. Any other claim needs to be justified.

Section 1: Compulsory for all

1. [4 marks] For the following problems, just write down the final answer.
   (a) Let F be the cdf of the standard normal, then what is the value of $F(4)$ rounded off to two decimal places?
   (b) A corporate board contains twelve members. The board decides to create a five-person committee to hide corporation debt. Suppose four members of the board are accountants. What is the probability that the committee will contain two accountants and three nonaccountants?
   (c) Suppose X is a random variable with $\Pr\{X = 2024\} = 1$. What is the variance of X?
   (d) If $M(t)$ is the MGF of a random variable, what is the value of $M(0)$?
   (e)
2. [3 marks] The following list contain random variables that occur in practice. *Indicate which distribution you will use* in each case. No explanations necessary.
   (a) *Total number of wins* in a series of identical games. Assume the outcomes of different games are independent of each other.
   (b) *The number of typing errors* made by a professional typist.
   (c) *The amount of time that elapses* between the breakdown of good machines.
3. [3 marks] State the weak law of large numbers and explain its significance in less than 50 words.
4. [3 marks] Let X, Y be two Bernoulli random variables with the following joint distribution:

$$\Pr\{X = 1, Y = 1\} = 0.3, \Pr\{X = 1\} = 0.5, \Pr\{Y = 1\} = 0.7.$$

   Compute $\Pr\{X = 0\}$.
5. [2 marks] State the law of linearity of expectations. State and derive the mean of a Binomial(n, p) random variable using the law.

Section 2

1. [2 marks]Three properties completely define a cumulative distribution function. State them precisely (in bullet points).
2. [2 marks] There are three boxes, one of them has two silver coins, another has two gold coins and the third has one gold and one silver coin. A box is chosen at random and the randomly drawn coin is gold.What is the probability that the other coin in the chosen box is also gold? Praphulla reasons as follows:"Once we see the drawn coin is gold, then the box with two silver coins is eliminated. So the chosen box can be one of the remaining two boxes. Therefore the probability is 1/2." Do you agree with Praphulla? Why or why not? Justify clearly.
3. [2 marks] Suppose X is a Gamma(2,2) distribution, compute the mode of X.

4. [2 marks] Let U be a Uniform($[0,1]$) random variable and let $V = U^2$. Compute the pdf of V.

5. [3 marks] A tool and die company makes castings for steel stress-monitoring gauges. Their annual profit, Q, in hundreds of thousands of dollars, can be expressed as a function of product demand, y:

$$Q(y) = 2(1 - e^{-2y}).$$

Suppose that the demand (in thousands) for their castings follows a pdf

$$f_Y(y) = 6e^{-6y}, y > 0.$$

Find the company's expected profit.

6. [4 marks] You have a fair coin. You toss the coin n times. Let X be the portion of times that you observe heads. How large n has to be so that you are 95% sure that $0.45 \leq X \leq 0.55$? In other words, how large n has to be so that

$$\Pr\{0.45 \leq X \leq 0.55\} \geq 0.95?$$

7. [4 marks] Boeing 757s flying certain routes are configured to have 168 economy-class seats. Experience has shown that only 95% of all ticket holders on those flights will actually show up in time to board the plane. Knowing that, suppose an airline sells 173 tickets for the 168 seats. What is the probability that not everyone who arrives at the gate on time can be accommodated? Approximate the answer using De Moivre-Laplace theorem. Justify all your steps. You may leave the answer in terms of cdf of the standard normal.

8. [6 marks] Suppose a life insurance company sells a Rs. 5,00,00,000, three-year term policy to a twenty-five-year-old woman. At the beginning of each year the woman is alive, the company collects a premium of P rupees. The probability that the woman dies and the company pays the Rs. 5,00,00,000 is given in the table below. So, for example, in Year 3, the company loses Rs. $(5{,}00{,}000 - P)$ with probability 0.00054 and gains P rupees with probability $1 - 0.00054 = 0.99946$. If the company expects to make Rs. 50,000 on this policy, what should P be?

Year	Probability of Payoff
1	0.00051
2	0.00052
3	0.00054

Section 3

1. [4 marks] Write down (don't derive) the MGF of a Gamma distribution and derive the formula for its variance using the properties of MGF.

2. [4 marks] If the standard deviation is less than 0.01% of the mean of a random variable, then I will say that the random variable is 'almost constant'. What is the minimum number of independent samples $X_1, X_2, \ldots, X_n$ that should be drawn from the Uniform($[0,4]$) distribution so that the average

$$\frac{X_1 + X_2 + \ldots + X_n}{n}$$

is almost constant?

3. [4 marks] Construct a pdf of a continuous distribution whose median is larger than the mean. Prove your claims.

4. [5 marks] A newly formed life insurance company has underwritten term policies on 120 women between the ages of forty and forty-four. Suppose that each woman has a 0.007 probability of dying during the next calendar year, and that each death requires the company to pay out Rs. 5,00,00,000 in benefits. Approximate the probability that the company will have to pay at least Rs. 15,00,00,000 in benefits next year.

5. [8 marks] Let X be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Suppose

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

Find the cdf of Y and plot the graph of the cdf.