REAL ANALYSIS - INTERNAL ASSESMENT - 1

(1) Check whether the following sets are finite / countable / uncountable. If the set is finite, list out the elements of the set and state its cardinality. If the set is countable (or uncountable), then exhibit a bijection with $\mathbb{N}$ (or the interval $(0, 1)$) respectively.

(a) $A = (-2, 2)$, the open interval in $\mathbb{R}$.

(b) $B = \{\sin \frac{\pi x}{10} : x \in \mathbb{N}\}$.

(c) $C = \{(x, y) \in \mathbb{N} \times \mathbb{R} : xy = 1\}$.

(2+1+2=5 marks)

(2) (a) Let (X, d) be a metric space and E be a subset of X. Define limit point of E.

(b) If x is a limit point of E, prove that every neighbourhood of x has infinitely many points of E.

(c) What are the limit points of the subset $\{\frac{1}{n} : n = 1, 2, \cdots, 10\}$ of $\mathbb{R}$ with the usual metric? (1+3+1=5 marks)

(3) Consider the set $T = [1, \sqrt{2}] \cap \mathbb{Q}$. What is the supremum of T? Prove it. (2 marks)

(4) Identify x-axis in $\mathbb{R}^2$ with the real line $\mathbb{R}$. Show that x-axis with the usual metric on $\mathbb{R}$ is a submetric space of $\mathbb{R}^2$ with the usual metric. (1 mark)

(5) Consider the subset $X = \{(x, y) \in \mathbb{R}^2 : y < 1\}$ of $\mathbb{R}^2$ with the usual metric. Check whether the following statements are true or false and justify.

(a) The limit points of X is the set $\{y = 1\}$.

(b) The interior points of the set are $\{(x, y) \in \mathbb{R}^2 : y < 1\}$.

(c) The set $X \cap \mathbb{R}$ is an open subset of $\mathbb{R}$.

(2+3+2=7 marks)