Internal 1
Linear Algebra

M.Sc. Data Science - Semester I
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Time: 90 minutes *Maximum Marks: 20*

1. (3 marks) Solve the following system using Gaussian elimination:

$$\begin{cases} x + 2y + 3z = 1, \\ 2x + 5y + 8z = 3, \\ x + 3y + 4z = 2. \end{cases}$$

   State clearly whether the system has a unique solution, infinitely many solutions, or no solution.

2. (4 marks) Let

$$v_1 = (1, 0, 1, 0), \quad v_2 = (0, 1, 1, 1), \quad v_3 = (1, 1, 2, 1) \in \mathbb{R}^4.$$

   Determine whether these vectors are linearly independent. If not, find a maximal linearly independent subset and extend it to a basis of $\mathbb{R}^4$.

3. (6 marks) Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x, y, z) = (x + y,\ y + z).$$

   (a) Find the matrix of T with respect to the standard bases.
   (b) Compute the range of T and determine its dimension.
   (c) Compute the kernel of T and determine its dimension.
   (d) Verify the rank–nullity theorem for T.

4. (3 marks) Prove that a linearly independent spanning set of a vector space is maximally linearly independent.

5. (4 marks) Prove or disprove: If P is a 3×3 matrix satisfying $P^2 = P$, then P must equal either the zero matrix or the identity matrix.